

PERMISSION TO ADD:  
MATH TEACHING LIMERICKS

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# MATH 101

Functions are just like numbers.  
They get lugged around like lumber.  
They get add- and subtracted  
and more interacted  
making us dumb and dumber.

(Cost Analysis)

The cost of producing is mixed.  
Part of that cost is fixed.  
And the rest, never-ending  
on level depending  
(or maybe on politics).

(Rational Functions)

Said your favorite Zada or Tante  
“for an asymptote horizonta  
“you need to use both  
“of the leading coeff’s  
“whether or not you wanta.”

(Exponential Functions)

The bottom is perfectly noble  
but  $x$  is now upwardly mobile  
so that constant, poor dear  
must downsize it’s career  
and it’s feeling a bit claustrophob-al.

These guys are as busy as beavers.  
They’ve got Monday morning fever.  
They just grow, grow, and grow.  
They’re NEVER zero.  
And they don’t remain one for long, either.

Do logs give you logarrhea?  
Is a logjam drawing near?  
Just use your credentials  
and take exponentials  
to make those ol’ logs disappear.

(Matrix multiplication)

Just pair off each row with each column.  
Take the sums of those products so solemn.  
Yes, beat the odds  
with those scalar prods  
or whatever the devil you call ‘em.

(Compound Interest)  
"Nothing succeeds like success,"  
as these formulas show with finesse.  
Thus again and again  
A gets bigger with  $n$   
though not quite as big as you'd guess.

(Annuities)  
And now here's another fine source  
(to tickle your brain in this course)  
of mo' and mo' dough.  
It's dough a go-go.  
Oh, don't you just wish it was yours.

# CALCULUS LIMERICKS

Diff-ing  $x$ -to-the- $n$  is fun.  
Change the  $n$  to  $n$  minus one.  
Then go back to  $n$   
and use it again  
in front of it all, and you're done.

We can diff any power of  $x$   
on this whole blasted campus of Drex.  
 $n$  can be miniscule  
as an H-molecule  
or as large as Tyranno Rex.

It is not my intention to vex.  
It is not my intention to hex.  
My only intention  
is merely to mention:  
The diff of  $x$ -square is  $2x$ .

It is not my intention to glare.  
It is not my intention to scare.  
My only intention  
is merely to mention:  
The diff of  $x$ -cube's  $3x$ -square.

There's a general rule for all this  
(in case someone pulls a pop quiz):  
 $n$  steps down two ways  
for the rest of its days  
and  $x$  remains right where it is.

"Than receive it is better to give."  
Also, "live, live, and let live."  
These are words to clutch  
but not as much  
as "velocity equals deriv".

(General Power Rule)  
Are you bored with powers of  $x$ ?  
What would you like to do next?  
Some powers of  $g$   
will do nicely  
but watch out for the special effects.

On, Dasher! On, Dancer! On, Prancer!  
Don't forget the power in the answer.  
It gives it might  
and makes it right  
also a little fancier.

Three cheers for parentheses!  
They make forests out of trees.  
But be careful about  
what's in and what's out  
or they'll look like they have some disease.

(Product Rule)

Don't forget -- f appears twice.  
g will behave likewise.  
If you make like a dunce  
and write them just once  
... well, you'll get what you get for half-price.

Said a wiseguy named Georgy O'Porgy,  
"Let's have a Quotient Rule orgy.  
"On top, to be nice  
"let g appear twice  
"and then on the bottom one more g."

(Chain Rule)

Rinky, dinky, dinky.  
g provides the link-y.  
But the x and the f  
are not to be left  
unless we run out of ink-y.

Ln x, you can't just be.  
You have to get diff'd, yessirree.  
So give ln the slip  
then take the recip  
to get one-over-x, easily.

(General Ln Rule)

On bottom goes the copy.  
And what goes on the top-py?  
Why, the diff, g-prime.  
It makes it rhyme.  
And please don't write it sloppy.

(Integrating Powers of x)

There's a general rule for all this  
(and it's something you don't want to miss):  
n moves up and down  
all over the town  
and x stays right where it is.

(Integrating Exponentials)

And now here's a grave admonition  
delivered with proper precision:  
It's about that k.  
It steps down just ONE way  
and the x doesn't go where it isn't.

Minus-one is a cool special case  
delicious and dainty as lace.  
So don't play the hero.  
Don't divide by zero.  
If you do, be sure to erase.

(Curve-sketching)  
We can plot and plot 'til we plotz.  
But we've got to plot the right spots.  
Or those lows and those highs  
could elude us like flies.  
Likewise the flips and the flops.

There was a young man named Kareem  
who explained, "For a local extreme  
"the tangent at a  
"to rest must lay  
"and we wish it the pleasant-est dream."

Said his kissin' cousin Trix  
"But not vice versa -- nix.  
"That tangent can flatten  
"as low as Manhattan  
"with no min, no max, just a mix."

A fair maid from North Minnesota  
was drawing a steep asymptot-a.  
When it got 'way too high  
she murmured bye-bye  
and mourned not a single iota.

(Integration by Parts)  
"We need f and g-prime," said Mitch  
"and it matters which is which."  
"But not to worry,"  
said his cousin Murray.  
"It doesn't work out, we'll switch."

Root-a-toot too-a-falutin'.  
It's time for some substitutin'.  
Take stuff on display  
and collapse it away  
right along with Leibniz and Newton.

The differential  
is essential.

(Area between Curves)  
If the graph of x-to-the-sixth  
with x-to-the-eighth is, not mixed  
only placed, for our practice  
on the same set of axes  
then subtract to find what's in betwixt.

Don't forget, those curves might cross  
in which case you should take time to pause  
so you don't subtract  
when add's where it's at.  
'Twould be a lamentable loss.

To split or not to split?  
'Til the question that stymied Hamlet.  
But when you're in doubt  
don't leave the split out  
though it's harder, I admit.

We could sit all prim and proper  
privileged as gold and copper  
but 'twould perk our careers  
if we'd get off our rears  
and become intersection-hoppers.

(Solids of Revolution)  
A strapping young woman named Evvie  
was handed a solid of rev-y  
and asked for the volume.  
She answered, quite solemn  
"it's not very big but it's heavy."

(Average Value of a Function)  
Riki tiki tavi.  
Here's some calculus savvy.  
The inteGRAL  
o'er the interVAL  
will give us our function's av-y.

(Integration Techniques: Sine-Saving)  
Said a technique freak named Zeek  
"If you think sine-saving is chic  
"and you want to save  
"yourself into the grave  
"try cos and tan and sec."

Here's a little ditty  
helpful if not cute:  
What we save is the deriv  
of what we substitute.

(Deriv's of Trig Functions)  
A darling named Clementine  
said, "First sine, then cos, then sine.  
"And the minus and plus  
"make things even wuss.  
"Can't it make up its mind?"

A Non-Limerick:

Can we integrate  $\tan$ ?

Yes, we can.

Can we integrate  $\secant$ ?

No, we can't.

(Trig-triggers -- i.e., trigonometric substitutions)

A trig-triggering trickster from Beacon  
is stuck on an odd-powered  $\secant$ .

An integral table

would render him able

but his conscious is prodding "no peekin'".

Trig, trigger, triggerest.

Try not to be too vigorous.

If you trig too much trig

it'll get too big

and you'll fall down just like Icarus.

(l'Hospital's Rule) (Good Golly, Miss Molly! Here comes l'Hospital-y.)

There was a fair maid from Nepal  
who was expert in ol' l'Hospital.

She diff'd 'til she dropped

on bottom and top

then murmured "Nice knowing y'all."

(Hyperbolic Functions) (Are you a hyperbolic-aholic?)

Take the trig I.D.'s, all kinds

add  $h$  to those  $\cos$ 's and  $\sin$ 's

and this derring-do

leaves them all still true

except for a few minus-signs.

(Completing the Square)

When the middle term nerdily lurks  
use this to get rid of the jerk.

Sometimes a wild guess

will get rid of the mess.

And algebra always works.

(Approximate Integration)

The Trapezoid Rule can be fun.

All those 2's will get the job done.

But watch out, my friends

for the left and right ends.

At those we will need only 1.

Cheer up. (Don't look like such wrecks.)

I know this seems pretty complex.

And there's many a  $y$

where the midpoints lie

but there's only one  $\delta$ - $x$ .



Just look at those sixes and fours!  
They're the prettiest things in this course.  
And this cool rule of Simpson  
will stay sweet and winsome  
once you've become sophomores.

(Improper Integrals)  
Said a ship-shape chap from the Congo  
"Since our region is infinite long-o  
"it's likely that you'd  
"be inclined to conclude  
"that it's infinite big, but you're wrong-o."

Said a dashing young bloke named Apollo  
"Said our region is infinite tall-o  
"It's likely you'll dig  
"it be infinite big  
"but sorry, that doesn't quite follow."

(Separable Differential Equations)  
A ditzo from hither or thither  
refused to get all in a tither.  
"Tis true," said he, "I  
"can't tell  $x$  from  $y$   
"but can't tell left from right- $y$ , either."

The vertical line test's a pity.  
It stops curves from being pretty.  
But we can make loops  
without saying "Ooops?"  
with parametric graph-iti.

(Polar Coordinates)  
With these we can also be arty.  
We can have a plotting party.  
While away the hours  
making petals and flowers  
along with Mercenne and Descartes.

Would you like to come up to the board  
and draw  $r$  and theta coord's?  
We'll get hulas and hoops  
and loop de loops  
and other delicious rewards.

(Infinite Series -- Comparison Test)  
Smaller than small is small.  
Taller than tall is tall.  
And that's how it goes  
and that's how one knows  
whether anything happens at all.

(Integral Test)

If we know what happens with n's  
all x's will follow, my friends.  
It's all essentially  
the same, consequentially  
and on each the other depends.

(Alternating Series)

However they rageth and roareth  
and wobbleth back and forth,  
you'll eventually find  
that they make up their mind  
someplace between south and north.

(MacClaurin Series)

The first in our fine repertoire  
is one over one minus r.  
We mustn't forget it  
for where we are headed  
or else we won't head very far.

An industrious lad from the near East  
was summing a long Taylor series.  
At the twentieth head  
he just shrugged and said,  
"That's as far as I go, my dearies."

(Partial Deriv's)

We x 'em and we y 'em.  
But not both at the same time.  
 $f_x$  is one  
 $f_y$ 's also fun  
though we never get to f-prime.

(End-of-Term Farewell)

Yes, I know, it's 10:49.  
But this proof needs one more line.  
I promise I'll write  
with all of my might.  
(Keep mum if I'm off by a sign.)

# COMPLEX ANALYSIS

A reality complex had Fred.  
He felt kind of bad in the head.  
So he took Complex Var  
with Cohen Mar  
got a complex complex instead.

There was a young lady named Suzie  
who couldn't add  $2z$  and  $2z$ .  
She said, "Can't it be  
"without that ol'  $z$ ?"  
She's being a little too choosy.

(nth Roots of Unity)  
There are  $n$  of them sprawled on a wheel.  
Among them at most two are real.  
The others must go  
half above, half below.  
But they get paired off in the deal.

$e$ -to-the- $z$  is exotic.  
And it changes a sum to a product.  
It has no root  
but it's kind of cute  
and it's vertically periodic.

A young man named Kenny macKenzie  
had trouble computing  $\ln z$ .  
He breezed through the  $r$ -part  
with the slickness of pop art  
but the  $\theta$ -part gave him a frenzy.

Here's the key to  $z$ -to-the- $c$ :  
It's just  $e$ -to-the- $c \ln z$ .  
But watch for  $\ln y$ .  
Its values are many  
though sometimes  $e$  makes them agree.

(Said Sin and Cos)  
"In trig and in calc, all through  
one was the limit we knew.  
But now we're set free.  
Now we can be  
anything we desire such as two."

How smart the conditions of C-R !  
They're almost as brilliant as we are.  
But they're known for their sly-ness.  
Watch out for that minus  
or we'll spend half the night in the E.R.

Yes, here they come, hot off the griddle  
spiffy and spicy and little.  
One sports a plus  
the other mi-nus  
and none of them sports the middle.

The research team Cauchy and Goursat  
was busting its brain and its torso.  
They went huffin' and puffin'  
but came up with nothin'.  
Now they're living on noodles and orzo.

The pretty professor from Jersey  
just stood there and pleaded for mercy.  
She said "in complex  
"v-y equals u-x --  
"or maybe it's vice vers-y."

i after e after u  
after pi after n after 2  
after p'renthesis ln  
and then no more spellin'  
unless we can find something new.  
(Thanks to Bob Blackard for getting me started on this one.)

Our pretty professor's a rarity.  
Her lectures the height of clarity.  
Except that when  
she does minus-one to the n  
n turns out to be the wrong parity.

The pretty professor, our rarity  
was engulfed in hyper-hilarity  
'cause again she had spun  
a spare minus-one.  
She said, "Well, we can give it to charity."

(About  $\oint f(x)/(z - z_0)^n$  )  
When C's end is at its beginnin'  
ask "Is z-nought out'n or in'n ?"  
Draw the point, draw the curve.  
Then stand back and observe.  
If in doubt get a second opin'on.

Computing it's easy as pie  
(especially if you try).  
There's a derivation  
and an exclamation  
and don't forget two pi i.

This classroom is water- and sun-proof  
and poison- and noisin'- and gun-proof.  
It's as pure as the Gospel  
of everything poss'ble  
except that it's not minus-one-proof.

Geometric ser's are a gem.  
But let's add that little m --  
OUTSIDE, if you please  
the parentheses.  
Did you hope I'd forget about them?

The last of those spunky Mohicans  
was at work on a long Cauchy sequence.  
It was i - pi - y  
from Mon to Fri  
and he took it home on the weekends.

Can you count? Then here's one for you.  
One less z to the minus-two.  
Write 1, 2, 3  
then blank, z, z  
and the powers and the plus-signs, too.

A fair maid from North Miami  
was at work on the triple-whammy.  
She tried subtraction  
and partial fractions.  
Then she called for her pappy and mammy.

A lazy young lad named Laurent-o  
used his series to integrate pronto.  
Just b-one will do  
and as for b-two  
you need only find that it you want to.

O what will the fair miad named Tessa do?  
She's been after a simple-pole residue.  
She says, "NOW I see.  
"It's q-prime UNDER p.  
"Then I plug in z-nought -- or I guess I do."

TEACH-TONE PHONE (Remember, I gave out my phone number? Well, no  
one ever called, but if you had, here's what my recording said -- See what you  
missed?)  
For Cauchy-Goursat  
press O.  
For some integral fun

press 1.  
For a residue  
press 2.  
For  $1 / (1 - z)$   
press 3.  
For contours galore  
press 4.  
But to find out the fate of a minus sign  
be patient, please, and stay on the line.

# LINEAR ALGEBRA

Allow me some words of great wisdom  
about any linear system:  
It is represented  
by a matrix, augmented  
which explains this course's existence.

Do row op upon row op.  
Keep doing them 'til you drop  
or until rectangular  
becomes triangular.  
(And the triangle's on top.)

The form which is just-plain row ech-  
means surely you'll still have to thresh  
with back substitution  
so keep on reducin'  
to get x-sub-i in the flesh.

(Matrix multiplication)  
Just pair off each row with each column.  
Take the sum of those products so solemn.  
Yes, beat the odds  
with those scalar prods  
or whatever the devil ya call 'em.

(Definition of Determinant)  
O, nothing could ever be finer  
than replacing A major with minor.  
Of course, for most n  
we must do it again.  
And the signs will get sign-er and sign-er.

( $\det A \times \det B = \det AB$ )  
I don't mean to cause you to squirm  
but the computations confirm  
that determ before prod  
(in the eyes of God)  
gives the prod before the determ.

When we wish that our fingers were toes  
and we wish that the columns were rows  
and the i's and the j's  
are turned the wrong ways,  
'tis time to bring on the transpose.

(how to find A-inverse)  
On cofactors get yourself versed.  
But start with the transpose first.  
And do not forget  
to divide by det  
and you'll wind up forever cursed.

If you know how to cut and to paste  
and can figure determ's with great haste  
and one more aside:  
if you've learned to divide  
then by Cramer's cool rule you are graced.

(to decide whether a given set of functions is linearly independent)  
Line up all the functions in sight.  
Then diff them with all of your might.  
Then sing a sweet song  
of right and of wrong  
as we get that ol' Wronskian right.

A young man named Timothy Tigen  
was searching for values eigen.  
But the characteristic  
and other logistics  
were such that he needed to try 'gain.

(how to diagonalize)  
E-values will get us big-D.  
E-vectors will get us homefree.  
Yep, string up the specs  
and line up the x  
and we'll get where we needed to be.

If its set of e-vectors is sizeable  
then A is diagonalizable.  
And the converse, too  
has been tried and true.  
(I hope that is all recognizable.)

(how to exponentiate a matrix)  
Proceed as the previous verse  
has instructed (for better or worse).  
Treat D as you're tempted  
and X is exempted  
except we'll still need its inverse.

(Definition of vector space)  
We plus 'em and muss 'em and less 'em.  
We shrink 'em and stretch 'em and press 'em.  
And still they will be  
inside our big-V.  
Such fine loyal souls, the Lord bless 'em.



(subspaces)

We can push and pull and stress  
and otherwise make a big mess  
and they'll not only be  
inside our big-B  
they'll also remain in big-S.

Can we get infinity from fin  
(without committing a sin)?  
Oh yes we sure can  
with the kind help of span.  
(After all, it's the past tense of spin.)

(dimension)

Each basis goes by its own name.  
Each vector goes by its own aim.  
But if you would count  
the number-amount  
the answers will turn out the same.

Base B can be used to express  
any member of V or of S.  
But the thigamabob  
that will pull off the job  
depend on which B we possess.

To get thigamabob for C  
from thigamabob for B  
will require the rendition  
of matrix, transition  
and seldom will they agree.

(linear operators)

They're linear, down to the core.  
They take zero to zero, no more.  
And sums and doubles  
and all other troubles  
to what they had been before.

(Multiplication by any matrix is a linear operator.)

Said a fair young maid named Alexis  
"Ax will double if x does  
and A takes a sum  
into something hum-drum  
and there won't be any more extras."

(rank)

A matrix has n column-vectors.  
Indeed, it's a vector collector.  
It's all touchy-feel-y  
but how many, really  
are there and how many are exter?

(Column rank = row rank)  
You can put them in lines or in layers.  
And then, if you say the right prayers  
it won't matter which.  
The quarter-turn switch  
won't change what's essentially there.

(Every linear operator = left multiplication by some matrix.)  
Just take what  $L$  does to each  $e$   
and line them all up merrily.  
And that's a good way  
to get your big- $A$ .  
(Watch out so you don't get a  $B$ .)

Let's all do a little Gram-Schmidt  
get orthogonal vectors that fit.  
At every stage  $k$   
I'll show you the way.  
(At  $k = n$  we can quit.)

# VECTOR ANALYSIS

Here's something we might contemplate:  
A vector's an arrow so straight.  
For forces, its length  
expresses its strength  
and direction expresses its fate.

Two vectors, whatever the size  
can be added, component-wise  
so make so this rhymes  
by adding three times  
then put in the k, j, and i's.

(Scalar Multiplicatoin)  
A spunky young woman named Ralpa  
would multiply vectors by alpha.  
She thought it quite nice  
to multiply thrice  
in this city of Philadal-pha.

(Scalar Product)  
Here's the most fun you ever had:  
Take products three times and then add.  
Yes, do mind your mommas  
and DON'T put in commas  
(or momma will tell your Dad).

(Vector Product)  
In this glossy and glorious firmament  
nothing's more perfect or permanent  
or more fancy-free  
for u criss-cross v  
than our ever-lovin' determinant.

i, j, and k are cute  
and they're specially fixed to compute.  
E.g., i cross j  
is lower-case k.  
but as arrows, they're too short to shoot.

(Some curve parametrization hints)  
Straight lines are quite matter-of-fact  
if you know how to add and subtract.  
True, there's multiplication  
in this situation  
but nothing on which you need act

Although we are all mere amateurs  
in this business of making parameters  
when it's circles in question  
I have a suggestion:  
Try not to forget the diameters.

(Formula for Curvature)  
r-dot dot r-dot-dot.  
Yes, sometimes we dot a lot.  
But dot-dot-dot?  
Or r-quad-dot?  
. . . Well, sometimes we do not.

For that curvature for t' compute  
that formula's really a beaut.  
But when figuring kappa  
do mind your Pappa  
and don't you forget that square root.

It's easy and fun to do grad.  
It makes us feel gleesome and glad.  
Don't you feel so alive  
when you partial-deriv?  
But put commas, now -- don't you dare add.

Here's something no one will object to:  
When doing the div please expect to  
takes partials, three  
of componenets of v  
and be careful which with respect to.

Aloha, tres bien, and shalom.  
May I mention a vital syndrome?  
Towards the end of the div  
you should get additive  
(or just wait 'til your father gets home).

Now, the hardest is curl, we agree.  
But let's have a determinant spree.  
First, three vectors little  
next, del in the middle  
and then on the bottom goes v.

All told (and telling it true)  
there's more in the curl to do.  
We've got partials six  
and note how they mix  
and the answer's a vector, too.

(Directional Deriv.)

If we're out in the wide blue yon  
rates of change depend, which directi-on.  
So divide b by mag  
then dot it with grad  
and we'll find out how quickly we've gone.

(Unit Normals to Surfaces Given in the Form  $f(x, y, z) = C$ )

Said a dashing young man named Aeneas  
"I've got a few dandy ideas:  
"Just take our f's grad  
"divide by the mag  
"and it doesn't much matter what C is."

(Reality check)

Whenn these poems you so dutif'ly edit  
keep one thing in mind (to my credit):  
I realize that mag  
does not rhyme with grad  
but the point is: YOU won't forget it.

(Surface Parametrization Hints -- Plane)

If you want to be this term's winner  
remember, the keyword is linear.  
So let us let z be  
ua + vb  
(for integrals outer and inner).

(Cylinder)

First, think of the xy-plane.  
(That's not such a drain on the brain.)  
Next, move once around  
and then up and down.  
If tired out, go hop on the train.

(Sphere)

Said a student from South Singapore  
"We use cosines and sines galore  
"or the sum of three squares  
"would give us nightmares --  
"Yes, that's what the trig stuff is for."

(Line Integrals)

And now we've got functions three.  
What could the integral be?  
The answer, essential:  
Component, tangential.  
It gets things in terms of small-t.

(Surface Integrals)

Next, functions (three) with a surface  
(And Murphy's Law gets more Murfish).  
But the answer, informal:  
Component, normal  
to get u's and v's at our service.

(Triple Integrals)

Now, triple int's can be quite chic  
so of them we'll now start to speak.  
But one thing clear:  
No vectors here  
(or you're grounded for a week).

(Divergence Theorem)

Next, the surface int. above  
that we've all come to know and love  
will be giving 'way  
on this fabulous day  
to a triple int. of the div. of.

(Stokes Theorem)

And now it is late, not early.  
'Tis time for Stokes' Theorem, surely.  
Any int. (closed line)  
can be redesigned  
as an int. involving the curl-y.

(Reality Check)

Again, as these lim'ricks you edit  
remember one thing (to my credit):  
I know that designed  
does not rhyme with line  
but it rhymes more than how Stokes said it.

# DIFF EQ

A fair maid from Northwest Virginia  
was solving a first-order linear.  
But that exponential int.  
was too large to print  
so she just made it shorter and skinnier.

A starry-eyed lad from Wisconsin  
was hoping those coeff's would be constant.  
But his hopes turned to hexes  
at all of those x's.  
He shrugged and remarked "Stuff and nonsense".

There was a young woman named Sally.  
Legendre was right up her alley.  
"Minus-x-square plus one  
"is what starts the fun  
"and n-square plus n's the finale."

Her cousin named Mary Magee  
said "Bessel seems besser to me.  
"x-square WITHOUT one  
"seems much more fun  
"and nu's cooler than n," said she.

Their cousin from Doodle-Skadiddle  
said, "Yes, but what's in the middle?"  
Answered they, "we suspects  
"it's x or 2x.  
"We're feeling a big non-committal."

An attractive young ms. named Cassandra  
was solving equations Legendre.  
a-nought was do-able  
a-one construable  
but a-(n+2) was beyond 'er.

There was a fine fellow named Ian  
who was ready to give up on  $p_n$ .  
He said, "Too much mess  
"with the n and the x".  
And sometimes I can't help agreein'.

Let's do  $J_n$  versus  $P_n$ ,  
The former's a much smaller bein'  
with much less subtraction  
and all told, less action  
'cept: more plus-signs in between.

We've  $y_1$  but not  $y_2$ .  
What're we gonna do?  
Just look to the board  
at Red. or Ord.  
how, instead of C, we try u.

We've  $y_1$  and  $y_2$  sublime.  
But r 's not zero this time.  
So do Var. of Par.  
with Cohen Mar  
then stand up and drink to l'Hyam.

(Picard's Successive Approximations)  
Again and again and again  
from n-1 to n.  
We get closer and closer  
(If it's cos, we get cos-er)  
but making no promises when.

(Euler-Cauchy Equations)  
They sport both dx and dy  
and an x-square and a by.  
And the thing to do next:  
guess a power of x.  
When you plug it in, you'll soon see why.

(Bessel Functions)  
In these guys the x appears twice  
'cause the powers get strategically sliced.  
The x that is mighty  
goes to the right-y.  
To the left goes the x that is nice.



## PDE

A lad thought he'd service society  
by studying math and psychiatry.  
But his plans went a-ragin'  
with the first wave equation  
'cause he got Separation Anxiety.

Four, Fourier, Fouriest --  
Isn't it just-plain glorious?  
How, from sin's and cos's  
we get (by osmosis)  
so much, and emerge victorious.

(Fourier Series in Exponential Form)  
Those functions e-to-the-in-x  
can be treated like cos x and sin x.  
And if they can't  
call me Bell Atlant-  
or try MCI or Ninex.

(Fourier Integral)  
Are we feeling all morbid and mopey  
just 'cause  $f(x)$  has no p?  
Don't be so timid.  
Go to the limit.  
When there's life, there's always hope-y.

It's Jan. 14 and our prof  
by a factor of TWO is off.  
Is she going to cry?  
Says she, "No, not I.  
"In fact, I'm more likely to laugh."

Do you think this is getting hum-drum?  
It's time to consider a drum-drum.  
We'll get double series  
and double theories  
and B's and B\*'s under 'em.

Next we make our rod very lengthy  
thus increasing its impact and strength-y.  
So we've lost the L  
but gained integrell.  
For thy patience, I humbly thank thee.

There was a spry guy from North Saigon  
who was searching for values eigen-  
But the boundary conditions  
surpassed all his wishin's  
and wouldn't let bygones be bygone.

Oh, trig functions bellow and bark  
and Bessel's go bump in the dark.  
but Sturm-Liouville creatures  
have all sorts of features  
and fill up the whole Noah's ark.

# ABSTRACT ALGEBRA

We're a bunch of nincompoops.  
We can't get a grip on our groups.  
We ask, in a frenzy  
"Is it  $Z_n$  or  $nZ$ ?"  
Guess wrong, so grin and go "Ooops".

This quarter we're all pretty group-y.  
It can get pretty meaty and soup-y.  
 $Z_3$  order 3  
 $Z_p$  order p  
and D-sub-p order 2p.

(You know how that religious mathematician Kronecker said "God created the integers; the rest is the work of man" -- Well . . .)  
"Integers are neat," said God  
"both the even and the odd."  
"So I'll give you some, Kronecker  
"for Christmas or Chanukkah.  
"Do you mind if I package them mod?"

Said a group theory pro named McClellan  
"We've too many isom and elem  
"and homom and autom  
"but already bought 'em.  
"Perhaps we could try to re-sell 'em."

(Automorphisms)  
Some're outer and some're inner.  
All're autom's (although it's winter).  
Some turn out to be  
the mere identity.  
But that's good when you're a beginner.

Rub-a-dub, rub-a-dub, bub-bub.  
What, pray tell, what, is this hub-bub?  
It's subgroups so cool.  
They obey the strict rule:  
Order-wise, they divide what they're sub ub.

In the kitchen you'll hear lots of slubbering.  
In the bathroom you might see a tubring.  
But the classroom, I claim  
is quite tidy and tame.  
All it's got is a subgroup or subring.

(More dialog between God and Kronecker)

"Integers are great," said God  
"and it's fun doing sum and prod.  
"But I've now got some notions  
"to tackle the quotients"  
and all except K oo'd and ah'd.

"It was nice dividing by bd.  
"But now I'm getting more greedy."  
Thus spake our Lord  
standing straight at the board  
writing Q-bracket-x and Q.E.D.

"One was good for a laugh.  
"And two was a treat for my staff.  
"But I've now got a hankerin'  
"to do some tankerin'.  
"I'll start with one-and-a-half."

"For seven long days labored I  
"with integers low, then high.  
"But now 'tis day eight.  
"Tis time to create  
"quotients and roots and pi."

Deep in waters hot, not tepid  
feeling rueful but intrepid  
our two creators  
and integer-traitors  
shrugged "Sorry, we just can't he'p it."

"Please forgive me," said Kronecker L.  
"I hope you won't send me to hell.  
"But x caught my attention  
"and x led to extension"  
and the rest we know only too well.

Yes, pity the genius and hero  
who fiddled, but not like Nero.  
He started with F.  
Now he's got nothing left  
for he sold his soul for a zero.

(Primitive Element Theorem)  
A fair maid from Alabam-y  
was given a double whammy.  
But she knew how to mingle  
so's to make it a single  
and ace the final exam-y.

(When is  $F(a)$  isomorphic to  $F(x)$  ?)  
If about  $a$  we have so spec's  
then  $a$  might as well be  $x$ .  
And  $e$  and  $\pi$   
are good  $a$ 's to try  
though it's not quite clear how it checks.

We might grow up to be tax-collectors  
or city or country inspectors.  
But Chap. 23  
ensures that we  
will never be angle-trisectors.

(God confesses to Kronecker)  
First I couldn't stop at ten.  
The I couldn't stop at  $n$ .  
Then  $Z$  and then  $Q$   
and the square root of two.  
But I've now come to  $C$ . Amen.

(Epilogue: by God and Kronecker)  
We work together like brothers.  
One creates, the other discovers.  
And Galleon writes.  
And Cohen recites.  
And we welcome any others.

# PROBABILITY

(Set theory)

Hippity hippity hoppity.  
A set can be built through a property  
with nouns and verbs  
and blubs and blurbs  
or any ol' thigamabob-ity.

A set can also be seen  
as an element-making machine.  
That set will consist  
of all s in some list  
with commas in between.

A fair maid from North Beelzabub said  
"I don't want to get us all upset  
"but if s in B  
"implies s in C  
"then B is of C a subset."

(The empty set)

Here comes phi, consisting of nada  
and neither your mada or fada  
can fill-er it up  
for lunch or for sup  
not even with air or with wada.

(Union and Intersection)

The union of 9th and Race  
is something we'd care not pace.  
But the intersection  
leads to detection  
of that fabulous Thai lunchplace.

(Universal Set)

Here is one more thigamajig  
that we'll need for this whole shindig.  
Yup, time for big-U  
too big to be true  
but not too true to be big.

(Complement)

Whatever set A hath not  
is what A-prime hath got.  
And their intersection  
defies detection  
but their union is a lot.

(Cartesian Product)

Let A be of women a set  
and B be of men; then we get  
from A-cross-B troubles  
all possible couples  
providing we know they're all het.

(  $n(A \cup B) = n(A) + n(B)$  )

Here's one more essential point:  
If A is from B disjoint  
then to give you, I'm glad  
permission to add  
at the risk of seeming flamboyant.

(  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$  )

And now here's another cool fact  
on which you might soon need to act:  
You still, my comrade  
have permission to add  
and orders to also subtract.

(to calculate  $P_{n,r}$  )

Write down n, and then when you're done  
write parentheses n - 1.  
The next thing to do:  
Write n - 2.  
Keep going, but not down to none.

(to calculate  $C_{n,r}$  )

Here's something that's redder than henna:  
our blood when computing  $C_{n,r}$   
and the sweat and the tears  
as we and our peers  
do cancelling more than we wanna.

A fair maid named Mary Maltese  
got her P's all mixed up with her C's.  
She said "I do figure  
"the P's must be bigger  
"but hey! there are no guarantees."

Said her kissin' cousin Muriel  
"I'll give you a little tutorial.  
"In case you've forgotten  
"the C's, on the bottom,  
"sport an extra r-factorial."

(Mississippi & Co.)

And now here's a new little game:  
permuting when some are the same.  
Permuting these some  
will feel sort of numb  
although there might still be some pain.

(Reality Check)

As these poems you so dutifully edit  
keep one thing in mind, to my credit:  
I realize that pain  
does not rhyme with same.  
But it will if you're less anal-headed.

(Binomial Formula)

I'm sure you've a loyal FOIL-er  
and I won't be a FOIL-er spoiler.  
But when  $n$ , for its britch  
gets too big, you should switch  
and be to this formula loyaller.

(Outcomes)

When you do an experiment-y  
the possible outcomes are plenty.  
And it's kind of fun  
to find out which one.  
(It's a good one, Deo volente.)

(Sample Space)

And now let us go with the flow.  
Take the set of all outcomes, just so.  
Yep, time for big-S.  
Yes! yes! yes!  
Don't you dare say no!, no!, no!

(Events)

Here's something we'll use infinitum:  
Events (We'll both say 'em and write 'em.)  
And to poet-ize:  
Size-wize, they lies  
between the two previous items.

(Another Reality Check)

As these poems you all superintend  
allow me to make some amends.  
I know that "they lies"  
is bad grammar, you-guys.  
But what's one s among friends?

(Probability at last)

We can now introduce big-P.  
It means probability.  
P of S is one.  
P of phi is none.  
In between lies P of E.



(Probability of Disjoint Unions)  
Said a dude with a shrewd attitude  
"If events doth each other preclude  
"then it's easy to tune in  
"the prob of their union  
"providing we're in the right mood."

(  $P(E \cup F) = P(E) + P(F) - P(E \cap F)$  )  
The above should inspire deja vu.  
Take the rule from 6.2  
change the n to P.  
It seems clear to me.  
I hope it seems clear to you.

(  $P(E') = 1 - P(E)$  )  
For the prob of E's compliment  
take the prob of that E-event  
subtract it from one  
and you'll be all done.  
(You can give it up for Lent.)

If the prob of each outcome you do know  
then by Jupiter!, Jove! and by Juno!  
the sum of them all  
will happen to fall  
extremely close to uno.

(  $P(E) = n(E) / n(S)$  )  
And now here's our big opportune  
to do what we've always been doin' --  
permute and combine  
but twice this time  
on this beautiful late-afternoon.

(One More Reality Check)  
As these poems you so dutif'ly edit  
keep one thing in mind, to my credit:  
I know that time  
and combine don't rhyme  
but I'm not overly poetic.

(Expected Value)  
First, the possible values collect.  
And then, to their prob's connect.  
The grand finale:  
Expected val.  
What else could we hope to expect?

(Variance and Standard Deviation)  
Here's a question a trible mischevious:  
Are the deviations devious?  
The answer, I'm 'fraid  
can't be bought at Rite Aid.  
Perhaps we can get it at CVS.

(Binomial Distribution)  
n trials, each a yes or a no.  
We want x yes's, just so.  
The prob of that mix  
involves n-choose-x  
and the p's and the q's also.

There's no need to moan and groan  
'cause it's easy to do Poisson.  
There's nary a care  
with Cohen Mar  
and less with Marion Cohen.

(Markov chains)  
And now let us make a mad dash  
-- a transition matrices bash  
with gobs and gobs  
of "throwing prob's"  
regardless of whether we catch.

To see how to get here from thereabout  
P is all that we care about.  
But we're gonna need v  
most definitely  
for info concerning the whereabouts.

(The Last Reality Check, promise...)  
As these verses you classify  
as to whether the rhymes apply  
you'll notice that mix  
does not rhyme with x.  
The same would be true of y.

(Some farewell wisdom)  
So now that we've had quite our fill  
here's a life-lesson from my quill:  
The prob's, woebetide  
won't be on our side  
but the possibilities will.

# STAT

(Intros)

I promise, the mean isn't mean.  
Nor the deviation obscene.  
Nor is any statistic  
overtly sadistic.  
It hurts, I'll give you morphine.

"The trouble with a kitten is that  
"eventually is becomes a cat."  
And the trouble with Probability is that  
eventually it becomes Stat.

(Summary)

Too big is a populace  
to get all the info in place.  
So we gather a sample  
(one that is ample)  
and infer 'til we're blue in the face.

(Data-Values)

You write them all down on a sheet.  
(You try hard to make them look neat.)  
Some will be qual-  
and some will be quant-  
and ne'er the twain shall meet.

(Reality Check)

As that last you so dutif'ly edit  
keep one thing in mind to my credit:  
I know qual- and quant-  
(although I might want)  
don't rhyme -- still, I'm glad I said it.

Continuous, maybe discrete  
in inches, in yards, or in feet  
they're numbers all  
some big, some small  
and some of them dare to repeat.

(for a Population)

The size is denoted big-N.  
The mean is small-mu, and so then  
for standard dev, sig-  
ma (little, not big)  
or so we would recommend.

(for a Sample)  
Small-n, for the much smaller size  
(How could we proceed otherwise?).  
Then take the wild guess  
of X-bar and small-s  
or so we would strongly advise.

(Histograms)  
The small x's, if you please  
are the values that big-X doth seize  
and the various y  
so high and so spry  
are the relative frequencies.

(A non-limerick)  
The less-than probs are truly a breeze  
for you and your TI83's.  
And the more-than probs, I'm happy to say  
are a mere subtract-from-one step away.  
And last but not least are the probs between.  
TWO steps away and pretty routine.  
So get it straight forevermore  
and you'll do just fine in Chapter 4.

Three cheers for the theorem of Bayes!  
We'll cheer for the rest of our days.  
We'll get P-D-T  
and P-T-D  
as we shout out our hip hp hoorays.

The theorem of Bayes is fun.  
And it's certain to get the job done  
with sensitivity  
and specificity  
and some help from P-D and one.

(Binomial distributions)  
There's one for each n and each p  
in this functional big family.  
n, for our files  
is the number of trials  
and p is the rel-frequency.

(to find  $P(X(n,p) = x)$  -- a non-limerick)  
Second VARS will get us flowing.  
0 gets us where we're going.  
n, a comma, p, another  
x -- and then it's almost over.  
Just make sure that you don't miss  
hitting end-parenthesis.  
And if you want to see this through  
ENTER is the thing to do.

(to find  $P(X(n,p) \leq x)$ )  
This is something you should love  
since it's exactly as above.  
Only once diverge the path:  
Instead of 0, ALPHA MATH.

(Normal random variables)  
The standard normal's a wonder.  
At the ends it's tossed asunder.  
Its mu is none.  
Its sigma is one  
which is also the area under.

The curve is an exponential  
but knowing that isn't essential.  
The 83+  
is enough for us  
to get a good job at Prudential.

(Non-standard normals)  
These curves are all wondrous, too.  
But they've got different sigma and mu.  
Mu is the head  
and sigma's the spread  
but the area's one, never two.

(Continuous random variables)  
It isn't the curves so grandee.  
It's the areas under, you see  
that do the fine job  
of giving the prob  
that X lies between a and b.

(Fuzzy Central Limit Theorem)  
Honest, this isn't a scam  
that the top of the histogram  
(and I do not err)  
for "most" X, yes sir  
is a normal curve, yes ma'm.

(T183+ Keystrokes for finding Cumulative Prob's) (Warning: Non-limerick)  
If this, instead of Earth, were Mars  
we might hit VIRTH instead of VARS.  
If Earth were square instead of round  
we'd arrow up instead of down.  
And if we meant "Return to Sender"  
what we hit would not be ENTER.  
If our goal were finding z  
the next four strokes just wouldn't be.  
And if we didn't know which z  
we wouldn't hit it, natur'ly.  
And if we want to poof this venture  
we'll forget to punch in ENTER.  
But since none of these is true

we'll do just fine with what we do.

(TI83+ Keystrokes for finding z when we know the cumulative prob up to z)  
This might be something we all love  
since most of it is like above.  
Just, if we think it's much too nice  
we'll arrow once instead of twice.  
And if we're masochists, indeed  
we'll do those four strokes we don't need.  
And if we didn't know which P  
we wouldn't hit it, natur'ly.  
And if we were intent-preventers  
we'd foget to punch in ENTER.  
All in all, it works out great  
with cause, effect, and help from fate.

(finding cumulative prob's of non-standard normals)  
( If you're wize / you'll standardize. )  
Step One. Just take x minus mu  
divide by the sigma. Step two:  
Dig into your pocket  
(or where'er you stock it)  
and with your TI rendezvous.

(Central Limit Theorem)  
If, ladies and fine gentlemen  
we take all the samples, size n  
then the set of their means  
fulfills all our dreams  
for n at least three times ten.

(Confidence Intervals for the Population Mean)  
We can never exactly find mu.  
That goes for approximate, too.  
But the more the dissent  
the more confident  
we can be; that's the best we can do.

(Calculating Confidence Intervals with the TI83+ )  
We start with STAT arrow-right twice  
and then chose the Z int device  
next, put in our data  
some this-a, some that-a  
and CALCULATE makes it precise.

(Hypothesis Testing -- H-sub-A / leads the way.)  
For this very significant lesson  
H-A is the thing to obsess on.  
H-nought is there  
more for the flare.  
It makes things a little more pleasant.

(Various "tails" )

"A one-l lama is a priest.

"A two-l llama is a beast.

"And I would bet a silk pajama  
"there isn't any three-l llama."

A one-tailed test is a bombshell.

A two-tailed test is something else.

And I would bet a treasure chest  
there isn't any three-tail test.

(The Test Statistic)

Though its various symbols encumber  
it really is only a number.

On top goes the gap  
on bottom more crap  
making us dumber and dumber.

(Using the TI83+ for Hypothesis Testing)

From our trusty TI83  
we squeeze out the value of P.  
If alpha is steeper  
H-A is a keeper  
unless further tests disagree.

Yes, trust our trusty one.

It says do, consider it done.

And if P is foremost

H-A is toast.

(We can eat it with cinnamon.)

We start with STAT arrow-right twice  
and then choose the proper device  
next, put in our data  
some this-a, some that-a  
and CALCULATE gets it precise.

Said a Prob/ Stat whiz named Ms. Bertie

"The CLT is less sturdy

"so instead of the z

"we use little t

"whenever our n's less than thirty."

Guinness had said that he cudent

so Gossett decided he wudent

and so little t

is more famous than he

and bears the proud name of Student.

"Difference between" implies two  
and that will affect what we do.

There should be two x

(on the screen and the text)

I think that's a pretty good clue.

Proportions are good to make friends with.  
No sigma or s to contend with.  
Just n and p-hat.  
It makes for good Stat.  
So that's what I've chosen to end with.

(Farewell)  
If the pain persists on morphine  
try an anti-histamine  
or caffeine, or saline  
or dentine, or codeine  
or an anti-statistics vaccine.



# TOPOLOGY

We all know the concept of set  
but not of topology yet.  
The latter's a class  
of the first, but alas  
there's more to the story than that.

E.g., it is quite necessary  
that they all be in  $X$ , solitary  
and that intersects, finite  
also be in it  
and union, arbitrary.

(Accumulation points)  
These are points which a set seems to touch  
but not, we observe, very much.  
They can get just as close  
as your eye or your nose  
but never as close as your guts.

(Definition of continuity)  
No epsilon-delta in this  
since numbers don't even exist.  
No fractions, decimals,  
infinitesimals.  
Instead, we must find a new twist.

Points jump from the old to the new  
as the sets they comprise also do.  
And so, in this scrimmage  
the  $f$ -inverse image  
of each  $V$  must be some  $U$ .

Tau from tau.  
That's the law.

(Base for a topology)  
Hee-haw! Hee-haw! Hee-haw!  
A base is a sub-class of tau  
whose various unions  
hit through and through  
though it itself doesn't at all.

(Sub-base)  
Rub-a-dub, rub-a-dub, dub.  
A sub-base is not always sub.  
But it is a base  
in certain ways  
so we welcome it into the club.

If  $X$  is discrete (tau not slight)  
we claim continuity bright  
for any old  $f$   
from  $X$  (on the left)  
no matter what  $Y$ 's on the right.

And here's something equally deft:  
If  $Y$  is discrete (not bereft)  
then open, we cite,  
is  $Y$  on the right  
no matter what  $X$  on the left.

(Homeomorphisms)  
Oh, bring out the violins  
for here's where the drama begins:  
Whither goeth the first  
(for best or for worst)  
doth goeth the second -- they're twins.

(Metric spaces)  
These spaces we truly adore  
'cause they provide numbers galore  
for us to grab onto  
whenever we want to  
(just like we used to before).

(Hilbert space)  
Square-summable sequences come  
with a metric involving square-sum.  
And it's isomet-  
to a proper subset  
which might seem surprising to some.

(first countable)  
This means something's countable, yes.  
But what? Well, each point must possess  
a base local  
that is countable  
(perhaps more than one, but not less).

(second countable)  
Again something's countable, true.  
But this time just ONE thing will do  
for this kind of space:  
one countable base.  
I promise, we will not need two.

(separable spaces)  
They're countable, in some weird sense  
'cause some countable subset is dense  
thus reigning them in  
agin and agin  
making things rather intense.

A fair maid named Mary Michelle  
was smitten with Heine-Borel  
and her fair cousin Dinah  
liked Borel-Heine.  
They got on exceedingly well.

A net is a sort of jet-set.  
There's no fish that it cannot get.  
With an epsilon-pole  
it reaches its goal  
and manages not to get wet.

(totally bounded)  
Hippety, hippety, hoppety!  
A set which possesses this property  
is finite, of sorts  
(by all sound reports)  
though by thinking a trifle sloppity.

(The Separation Properties --- Seeya later, separator...)

Heigh-ho! Heigh-ho! Heigh-ho!  
Three cheers for our trusty T-oh.  
It separates points  
all over the joint --  
really just one of them, though.

T-one is a nifty go-getter.  
It's a tasty trusty trend-setter.  
Yep, it's very hep  
the way it doth sep-  
and it separates them better.

Yahoo! Yahoo! Yahoo!  
It's time for our trusty T-two.  
It deftly gets  
TWO open sets.  
Yes, that's what we trust it to do.

(regular spaces)  
Here comes a whole different name  
playing a whole different game.  
It dares to make bets  
on points and closed sets  
putting the others to shame.

(normal)  
This guy does a little duet.  
It can take on any two sets  
that are diagnosed  
as being closed.  
It's the best separator yet.

## SEPARATION STEW

We learned T-oh  
to and fro.  
And then T-one  
was lots of fun.  
We did T-two  
fro and to.  
Went through T-three  
with glitz and glee.  
And T-three-and-a-half  
gave us a laugh  
until T-four  
walked in the door.

I'm sure T-five  
would really jive  
and with T-six  
we'd get our kicks.  
Lucky T-seven  
would be pure heaven.  
Likewise T-eight  
would be just great.

T-alept-null  
would be wonderful.

But we stop at four.  
There is no more.

(In any normal space, any two closed sets can be “separated by a continuous function”.)  
How very superb and eclectic  
to deal with this layout so hectic.  
With mere open sets  
cool Urysohn gets  
all these numbers with nary a metric.

# CATEGORY THEORY

(Introduction: "Functions" in the Raw))

O hear ye! O mathematician!  
Let us make the momentous decision  
to give up  $x$  and  $y$ .  
We can if we try  
and still get to keep composition.

("Arrows only")

Now, objects are nice and compact.  
They stay put and concise and intact.  
But the stars of our show  
are those things that go  
from one to the other and back.

(Discrete category)

What's wrong with this party?, pray tell us.  
No one talking to anyone else.  
They stand in their corners  
like old Sunday mourners  
conversing with only themselves.

(poset categories, non-discrete)

This party's a bit more grandiose  
sporting conversation verbose.  
Just one way, true  
and one line, too  
but at least that one line isn't closed.

(Larger categories)

Uh-oh, I am getting a hunch  
that somebody's spiking the punch.  
That the gang is all here  
is most crystal clear  
and it's up to the raunchiest stunts.

There once was a chap from Milano  
who couldn't tell epi from mono.

I told him "get wize  
"and just memorize"  
but I'm willing to bet he's not gonna.

(Duality)

Hippity hippity hop.  
Ev'ry  $C$  has its own  $Op$ .  
But off  $C$ - $Op$ - $Op$   
both  $Op$ 's we can drop.  
Yop, after one  $Op$  we can stop.

(Universal mapping properties)  
O, what are little UMP's made of?  
And what are big UMP's made of?  
Arrows galore  
and much much more  
and that's just what we were afraid of.

(Product = projection-collection)  
This hunter shoots many a spear  
but just one into each object here.  
And any who dare  
to likewise fare  
must take a quick step to the rear.

(Co-product = injection-collection)  
It's the object of this grand old hunt.  
(Don't worry; the arrows are blunt.)  
And if anything tries  
to fare likewise  
it must take a quick step to the front.

(Are you an equalizer-sympathizer?)  
Its claim to fame is plain.  
Its aim is to tame the twain.  
Whatever the arrows  
how wide or how narrow  
so long as both ends are the same.

(Equalizers and co-equalizers)  
We can't get these kids to agree.  
They fight like Mohammed Ali.  
But between Dad and Mom  
on both sides, so calm  
methinks we can tame the twee.

(Pullbacks and pushouts)  
If one end is not the same  
we can still pay the game, I proclaim  
for a similar deal  
reinvents the wheel  
and we would be fools to complain.

(Limit)  
It's next in our grande repertoire  
shooting arrows so near and so far.  
And further, t'boot  
they have to commute  
with the arrows that already are.

(Products and equalizers get us all limits.)  
We don't need pullbacks or initials  
or ceremonies judicial.  
All we will need  
to finish the deed  
and all we will ask  
to accomplish the task  
and all we request  
to pass the test  
and all we require  
to light our fire --  
yes, all we need seek  
are prod's and eq's.  
So on we will plod  
with eq's and prod's --  
to make the whole thing official

(Functors)  
Here they come, marching in droves.  
Both the contrav's and the cov's.  
Some imbed  
and some forget  
and some we don't yet even know of.

(Adjoints:  $\text{Hom}(FC, D) \cong \text{Hom}(C, UD)$  )  
It's Old Hom Weekend, it seems.  
And in fact there are TWO Hom teams.  
But, except in name  
those teams are the same  
if we take matters to extremes.

(F and U)  
Things would turn out mighty rotten  
and F would feel pretty downtrodden  
and most disappointed  
because un-adjointed  
if forgetful meant also forgotten.

(Seeya later, Yoneda)  
Any C can be co-completed.  
Small-y is all that is needed.  
And if there's a goof  
somewhere in our proof  
don't worry, we'll simply delete it.

(Endo-functors and P-algebras)  
Endo's are sprightly as minnow  
and they endo where they doth begin-o.  
Their activities buzz  
with P-algebras.  
Get ready for that to contin-o.

(Finale)  
See, there's life after x's and y's.  
Do you savor the fun and surprise?  
Do you like how it works?  
And relish the perks  
like no compound fractions or pi's?

## FERMAT'S LAST THEOREM PROVEN

Fermat said the proof was too large  
to fit in the right or left marg-  
True, back of the paper  
or proof made to taper  
might help, but he said, "I'm in charge".

Now, Wiles didn't mind paper waste.  
In fact, it was true to his taste  
to use up whole reams  
to realize his dreams  
and he crossed out instead of erased.

Fermat was all snickers and smiles  
as he smugly stayed clear of the aisles  
and he thought "they'll be glum  
"but that proof will succumb  
"though it's going to take quite a-Wiles".

## A COMPUTER NON-LIMERICK

A one-r erol is a strain.  
A two-r errol is a pain.  
But we would be in greater peril  
if there were a three-r errol.