PERMISSION TO ADD:

MATH TEACHING LIMERICKS

Marion Cohen

PRE-CALC

TYPES OF FUNCTIONS

They come in all sorts of disguises
in the various cool Exercises.
Some ordered pairs
some tables (not chairs)
and some graphs with their x's and y'ses.

DEFINITION OF DOMAIN OF A GIVEN FUNCTION

Some x's the functions can play
and others it can't, nay nay.
Take all of the can's
and include in the plans.
Save the can'ts for a rainy day.

FUNCTIONS AND THEIR GRAPHS, GRAPHS AND THEIR FUNCTIONS

Functions have various drapes
and graphs have their various shapes.
We have a preponderance
of correspondence
from which nobody escapes.

LINEAR FUNCTIONS

The simplest graph is straight onto.
And what does it correspond to?
Why, ax + b.
'Tis easy to see
unless, of course, we doesn't want to.

SLOPE-INTERCEPT FORM: y = mx + b, m - slope, b =y-intercept

Some statements that might help us to cope:
Before x is written the slope
and after the plus
(without too much fuss)
goes the y-intercept (so we hope).

POINT-SLOPE FORM: y - y-one = m (x - x-two), has slope m and passes through (x-one, y-one)

The m says how much it doth lean
and as for the rest of the scene
we've got, just for fun
x-one and y-one
with a comma in between.

point-point: y - y-one = [(y-two - y-one) / (x-two - x-one)] (x - x-one) passes
    through both (x-one, y-one) and (x-two, y-two)
Two points doth this straight line pass through
(w,y) sub-one and sub-two.
Conversely, these four
(in this math so pure)
determine what's what and who's who.

PIECEWISE FUNCTIONS ("different strokes / for different folks")
There's more than one rule, woe-betide.
By which rule should we abide?
That depends on x
so stand back for two sec's
then do what that x is beside.

QUADRATICS
A fair maid from Indianapolis
was expert on drawing parabolas.
She extended their arms
and showed off their charms
and they looked absolutely fabulous.

SIMPLIFYING COMPLEX FRACTIONS (FRACTIONS WITHIN FRACTIONS)
Take the little denom's en masse
then their lcm, with pizazz.
It'll cancel all
to get rid of the small
but not of the big (alas).

RATIONAL FUNCTIONS: DENOMINATOR ALERT (Are you a denominator-hater?)
As fall months march on towards winter
(and our evening squints become quint-er)
watch out, this fine autumn
that the x on the bottom
won't make it unfit for gozinta.

TRANSLATING A LINEAR SYSTEM INTO A MATRIX
Erase every x and each y an'
each plus and each equal sign.
Be sure to take care
with each missing var
in line after line after line.

n EQUATIONS, n UNKNOWNS, UNIQUE SOLUTION
The rref will be nice and tidy.

On the left, an x-n I.D.
With tons and tons
of zeroes and ones
and the answer on the righty.

A ROW (OR TWO) OF 0'S (0 0 0 0 0...)
This row is the ze-ro row.
It's a row which gives no info.
And it will not budge.
You be the judge --
it says no no no no no.

NO SOLUTION, "BAD ROW" (0 0 0 0.... 1)
This row says more is less.
Can't make up its mind, I guess.
It says right is wrong
and short is long
and no no no no yes.

TELLING THE CALCULATOR TO PUT A GIVEN MATRIX INTO REDUCED ROW ECHELON FORM: A NON-LIMERICK
Second x to minus one
is how we get this thing begun.
And then to get a further start
two arrows right will do their part.
Next, lest our poor T-8-3-plus
get some impression wrong from us
we must type in how many rows.
How many columns also goes.
And then the numbers, ends and center,
separated by each ENTER.
Now 'tis time in this fine ode
to take a pause with Second MODE.
And then -- its second grand appearance --
sec x min' one's interference.
Arrows right and down (not left)
enough to get us down to rref.
Another ENTER -- Got that done?--
and one more sec x minus one.
(I promise that was the last time.
It's honest and it makes it rhyme.)
Now, one more ENTER -- you can't miss --
and then a close-parenthesis.
Now one more keystroke from your mentor:
big fat joyous final ENTER.
Now we're done (unless one checks it):

Not a keystroke but can EXIT.

Functions are just like numbers.
They get lugged around like lumber.
They get add- and subtracted
and more interacted
making us dumb and dumber.

(Cost Analysis)
The cost of producing is mixed.
Part of that cost is fixed.
And the rest, never-ending
on level depending
(or maybe on politics).

(Rational Functions)
Said your favorite Zada or Tante
“for an asymptote horizonta
“you need to use both
“of the leading coeff’s
“whether or not you wanta.”

(Exponential Functions)
The bottom is perfectly noble
but x is now upwardly mobile
so that constant, poor dear
must downsize its career
and it’s feeling a bit claustrophob-al.

These guys are as busy as beavers.
They’ve got Monday morning fever.
They just grow, grow, and grow.
They’re NEVER zero.
And they don’t remain one for long, either.

DEFINITION OF LOG
Do logs give you logarrhea?
Is a logjam drawing near?
Just use your credentials
and take exponentials
to make those ol’ logs disappear.

TWO BASES FOR LOGS
Two numbers have we to discuss
-- e and 1-0, just for us.
The first is a natch
the second a catch
and they're both in the 83-plus.

LOGS OF PRODUCTS; ln MN = ln M + ln N
The product can change to a sum.
But 'tisn't entirely humdrum.
There's an extra ln
(one for M, one for N)
and you know where that had to come from.

LOGS OF QUOTIENTS: ln (M/N) = ln M - ln N
From divide to subtract we can whittle.
But again, there's s smidgeon of diddle.
ln's will appear
in the front and the read
with the minus in the middle.

LOGS OF POWERS: ln (m^N) = N ln M
That N is too up for our taste.
We'll make it step down in great haste.
But be sure, as you slide
that that N lands outside
the ln, or it must be erased.

CHANGE OF BASE FORMULAS: LOTS OF LOGS
Oh, how can we deal with this b?
It's neither a ten nor an e.
Just spring into action
by making a fraction
with logs up and down, yesiree.

SOLVING EQUATIONS LIKE 9^(2x-1) = 27^(x+1)
Such a pile of glop and slop --
how can we make it all stop?
Just play the game
make the bases the same
then compare what we've got on top.

LOG-TRIGGERS: SOLVING EQUATIONS LIKE 3^(2x-1) = 5^(x+1)
Here's how to accomplish these missions:
Take logs, change the powers' positions.

You'll get rid of the ninny
and acquire a linny
with weird looking coefficients.

SOLVING EQUATIONS LIKE ln (x+1) - ln x = ln 2
We want just one log for each side.
So by Rule #1 we'll abide.
We'll clear up the fog
by canceling log
much to our supreme joy and pride.

TRIG LIMERICKS: TWO NEW ANGLES ON ANGLES
(1)
Rinkity dinkity dink.
More angles exist than we think.
And some wrap around
(without making a sound)
as though they had too much to drink.

(2) RADIANS
It's my supreme pleasure today
to one other measure convey.
The numbers are spare
but they do get you there.
6 goes around most of the way.

A lady name Katy O'Grady
was good at converting to radi-
ans. "Simple", said she
"just take the degree
"and times it pi over 180."

Her cousin named Mary Magee
knew how to get back to degree.
"Just multiply by
"180 o'er pi.
"My cousin, I'm sure, will agree."

MORE TRIG
Let's hereby consider our data:
x, y, r, and our angle big-theta.
It feels pretty blurred
to the point of absurd
but we'll get it straight sooner or later.

MNUEMONIC DEVICE/ sin theta = y/r, cos theta = x/r

Y-in' rhymers with sine
(at least if it keeps on tryin').
Does x rhyme with cos?
Nope, not even close.
(But maybe it will sometime.)

INVERSE TRIG FUNCTIONS (Hark, hark, the arc!)
We can go from x to sin x
(or even to 9 sin 9x).
But hey, can we play
the opposite way
with our calculators and Timex ?

LINEAR REGRESSION: THE KEYSTROKES (START WITH STAT.): a non-limerick:
STAT ENTER is the way to start.
It gets us L1, if we're smart.
Now list the x's, one by one.
(There might be many, but it's fun.)
Next arrow-right (get to L2)
then list the y's, that's what to do
(as many y's as there are x'es.
-- takes us all the way to Texas)
Don't forget, do what you mean
and hit the ENTER's in between.
Now, 2nd MODE is our next bet.
It gets us where we want to get.
STAT arrow-right and then a 4.
'Twill get us what we need, and more.
(Yup, a and b and r, r-square.
The whole nine-yards, extraordinaire.)
And now we're done -- Farewell, Godspeed!
-- unless, perchance, we hap to need
to view a dandy scatterplot
or use the function we just got.
To do the former in this sequel
ENTER last, first sec- Y=.
For the latter, I have reckoned
the reverse, Y= 2nd
followed by MODE VARS right-arrow
ENTER ENTER straight and narrow.
Next we hit the number that
we want to find the function at.
Then ENTER once again, that's it
to keep us satisfied and fit.
And now we're done. I'm not pretending.
Really truly 'tis the ending.

(Matrix multiplication)
Just pair off each row with each column.
Take the sums of those products so solemn.
Yes, beat the odds
with those scalar prods
or whatever the devil you call ‘em.

(Compound Interest)
“Nothing succeeds like success,”
as these formulas show with finesse.
Thus again and again
A gets bigger with n
though not quite as big as you’d guess.

(Annuities)
And now here’s another fine source
(to tickle your brain in this course)
of mo’ and mo’ dough.
It’s dough a go-go.
Oh, don’t you just wish it was yours.

LIMERICKS (AND OTHER CORNY POEMS) FOR MATHEMATICAL CONCEPTS I

arranged by section numbers in the text, The Heart of Mathematics, Berger and Starbird)

(WELCOME)

This first poem is just to say hi.

I'm glad you had time to drop by.

You might not like math

but you do like to laugh

and I promise, I won't make you cry.

(Chapter 1 -- HI DIDDLE DIDDLE, HOW 'BOUT A RIDDLE?)

Riddles are all fun and frolic

and some are of cool math symbolic.

We've seen them before

we'll see them some more

'til we become riddle-a-holic.

Just one stone is fancy and foreign.

The eight others, weighing less, aren't.

We have just two scales.

After one use, each fails.

Now, isn't that wholly abhorrent?

It’s a most dark and stormy night.
There’s no sun or sign in sight.
Along comes a native
all truth or lies made of
and he won’t tell us which, for spite.

To stick to your guns has appeal
but what about Let’s Make a Deal?
Should we switch to Door 2
or be faithful and true?
Which gets us the automobile?

We wish we could have our druthers
and choose both doors, northers and southers.
But when all’s said and done
if we’re faithful to one
we’re unfaithful to the others.

(Dodgeball)
This game involves X and O.
But it isn’t like Tic Tac Toe.
One board is a square
used by the first player.
The second gets only a row.

(Section 2.1:THE PIGEON CONTINGENT)

n holes are waiting in trees

n + 1 birds in the breeze.

So what say we, hence

is the consequence?

Shall we ask the Board of Trustees?

According to most religions

there can’t be infinity pigeons

nor infinity holes

to shelter those souls.

Still, we might find the notion bewitchin’.

In the midst of this pigeon contigent

along comes another pigeon.

But no doom and gloom.

We can make room.

Just ask each to move over a smidgen.

I don’t mean to be all that stringent

but here come infinity pigeons.

Still, we can make do.

The trick involves 2

and a tidbit of twitchin’ and switchin’.

(Section 2.2: FIB LIB)

(non-limerick, after Lewis Carroll -- “Beautiful soup, so rich and green
waiting in a hot tureen.
Who for such dainties would not stoop?
Soup of the evening, beautiful soup.”)

Fabulous Fibs, so wholesome and true.

Each is the sum of the previous two.

Delicious as ice cream and Chinese spareribs.

Fibs of the evening, fabulous Fibs.

(limericks again)

All hail to our friend Fibonacci.

He tinkered like ol’ Liberaci.

He started off slow

two one’s in a row

but ended up vivo vivace.

The prolific Professor Gibbs

got a Ph.D. in Fibs.

It was Fibs upon Fibs

and they stuck to his ribs

but don’t worry, that’s not all he did.

Fibs are the latest fad.
And we're very faddish, and glad
to dribble out Fibs
no trials 'n' no tribs
as long as we know how to add.

(About the quotients, F-sub-n+1 / F-sub-n)
The F-sub-n grow like a tree
but those quotients are not quite that free.
True, they wobble and whittle
but only a little
as they zero in on phi.

(more Lewis Carroll)
Feelgood phi, so misty and mystic
yet mathematically realistic.
Royal and golden as a queen bee.
Phi of the evening, feelgood phi.

Now, according to mathematics
phi's a root of an easy quadratic.
It's quite irrational
so not computational.
Still, it works for phi-phanatics.

Phoebe is feeling phi-fever.
She's a passionate phi-believer.
And she's rather go-gettish
about that phi-fetish.
She's a fiendish over-achiever.

Her cousin Fifi feels iffy.
She's not quite sure phi's all that spiffy.
But connection to Phoebe
turns phi into freebie
so she beats that ol' phi in a jiffy.

The querellous Queen of Shebie
howe'ver rich and famous she be
-- She's the Queen of queens
but can't do phi for beans.
She should contact Citizen Phoebe.

(Quadratic Formula-la-la)
The bottom is tiny and terse
but sorry, the top is much worse.
Minus b, for a start
but alas, the next part
is too long to include in this verse.

And now we'll continue the answer.

Square root sign kicks off the bonanza.

Then square of b

minus 4ac

and we won't need another stanza.

Yatata yatata yaddam.

Don't forget the 2 on the bottom.

For if you do

forget that 2

you'll miss what you've forgottem.

Hippity, hippity, hop.

Don't forget the 4 on the top.

If you ignore

that vital 4

you'll miss what you forgop.

Our professor so mathematic

is using the formula quadratic.

She says "Oh, I see --

"it's MINUS b.

"Oh, why are these things so erratic?"

(a Quadratic Formula non-limerick -- It's all here!)

When a x-square plus bx plus

c equals zero threatens us,

no need to in self-pity wallow.

All you've need is soon to follow:

x a fraction, draw that line

and make it long (and not too fine).

Now, on the bottom (piece o' cake)

just write 2a (for old time's sake).

And on the top, though not as short

comes stuff not hard, if you're a sport.

Here minus b begins the jaunt.

(Please take as much time as you want.)

Then plus-or-minus next to do:

the square-root sign. (Make that long, too.)

And now that famous ol' discrim'nant.

(Yes, you knew that that was imm'nent.)

b again, but this time squared

then minus-sign (but plus we're spared).

Now 4ac (was that a mouthful?

But you're young, and feeling youthful.)

That is all -- was that so bad?

You're heroes like Sir Galahad.

MORE ALGEBRA LIMERICKS

SQUARE OF THE SUM OF TWO NUMBERS

Heigh diddle heigh diddle diddle
Don't forget the term in the middle.
If you do forget
you just might regret
(though probably only a little).

'Tis a sad fact of life, and intense
but one which makes good common sense
that a prod. to the nth
gets us rid of parenth-
eses, but it brings on more n's.

(Section 2.3: THE PRIMAL SCREAM)

A whole lot of numbers are prime.
Up, up, and up they climb.
Indeed, there's no end to them
so math people tend to them
all over space and time.

A whole lot of numbers are not prime
but every number has got primes.
Yes, each is a product

of primes so exotic.

(This last line will almost but not rhyme.)

The cantankerous Mr. O'Grimes

was a whiz at discovering primes.

2, 3, 5, and 7

211.

But he wants to find more primes that rhyme.

(more Lewis Carroll)

Such pretty primes, so basic and pure.

None is the product of anything more.

The favorite sport of Mr. O'Grimes.

Primes of the evening, such pretty primes.

(Section 2.4: THE MOD SQUAD)

(The following limerick is by John Ward McClellan.)

A lady of 80 named Gertie

had a boyfriend of sixty named Bertie.

She told him emphatically

that viewed mathematically

by modulo 50 she's thirty.

(back to my own limericks)

A pixie of sixty named Wendy

had a boyfriend of fifty named Kentie.

She told him emphatically

that viewed mathematically

by modulo 40 she's twenty.

(limerick on the test -- Fill in the blank.)

A heavenly seventy, Kate

had a boyfriend of fifty named Nate.

She told him emphatically

that viewed mathematically

by modulo _____ she's 28.

The mod part is what we divide by.

Remainder is what we abide by.

And the quotient, poor dear

... well, its only career

is to show up and then to go bye-bye.

(Check-digits)

A young lad from Southwestern Phoenix

raised his hand for that question ‘bout Kleenex.

When a slip of his pen

led to nine mod ten

he asked “Am I off by 16x?”.

(Section 2.5: N- AND D-CODING, using Fermat’s Little Theorem)

Computers bask well in their glory

and everything’s quite hunky-dory

when the task assigned

is multiplyin’

but factoring’s a different story.

(Fermat’s Little Theorem)

In this formula p appears twice

and so does the 1, which is nice.

But a, the poor dear

is doomed to appear

only once, but that still will suffice.

(Cohen's Slightly Bigger Theorem)

p and q each appear twice

and 1, lucky duck, appears thrice.

But once again, a

does not have much say.

How's that for pneumonic device?

(Section 2.6: Irrational Numbers)

A perky young damsel named Freda

was an adamant fraction-hater.

Well, for people like her

good news will occur:

The number of non-fraction's greater.

(Clique-y Rationals)

We can add 'em, mult 'em, divide

again and again, far and wide.

Yes, arithmetic

keeps them in the clique.

It's not easy to kick them outside.

(Lewis Carroll again)

Sweet little rationals, p over q.

Probably no big surprise to you.

Still, to me they seem most sensational.

Rationals of the evening, such sweet rationals.

Spooky irrationals, roots or worse

permeating the universe.

They’re citywide, statewide, international.

Irrationals of the evening, spooky irrationals.

(Spooky Rationals -- Getting into the Club)

The square root of two plus the square

root of two... keep going from there.

With all those roots

being in cahoots

things seem hopeless, but don’t despair.

Rationals are far too clique-y.

And irrationals too tricky.

In all of this dinn

it seems we can’t win.

But we’ll look them up on Wiki.

(Sect. 2.7: THE NON-MINIMUM CONTINUUM)

Numbers are squashed as can be.

They're locked in without a key.

Howe'ver in we zoom

there's no elbow room.

Nor is there room for a knee.

(.13131313...keep-on-going = 13 / 99 -- rational)

There's more of the long than the short of it

and we might soon begin to get bored of it.

But surprise! surprise!

Fraction-wise,

I promise, we'll reap the reward of it.

How long since you did long division?

I hereby am giving permission.

Yes, take any fraction

and get into action

then reap the reward of your mission.

(A non-limerick: .99999.... = 1 Also, .253999999..... = .2540000000... Etc.

Two different decimals, two different kinds.

One's got those zeroes, the other those nine's.

How could they possibly turn out the same?

be the same number with two different names?

(Chapter 3: InfinitIES / if you please)

Let's talk one-to-one correspondence --

indeed that's what measures abundance.

Yes, that is the name

of the numbers game

though you won't ever find it on Sundance.

A line is a line is a line

said the great poet Gertrude Stein.

Well, SHE said "rose"

and so I suppose

I can claim THIS poem as mine.

A line is a line is a plane.

Now, that's GOT to be purely insane!

It would not be fine

with Gertrude Stein

so I'll claim it as mine again.

A line is a square is a cube.

Don't expect to find THAT on U-Tube.

Nor would it be

on MTV

or the poems of Ms. Gertrude.

But a line is not a dot

not even a lot of dots.

Yes, you could jot

dot after dot

but you'll always miss a spot.

Infinities start out with vigor

and they get even bigger and bigger.

Yes, please don't get upset

but the set of subsets

is an even larger figure.

(a non-limerick)

Two crews, two coups, two shoes, two ewes.

It doesn't matter. All are two's.

Three bees, three fleas, three trees, three keys.

It doesn't matter. All are three's.

Four doors, four drawers, four floors, four stores.

It doesn't matter. All are four's.

n hens, n wrens, n dens, n glens.

It doesn't matter. All are n's.

The list of evens, list of odds.
On they plod in grande promenade.
The list of whole numbers, neg and pos.
List of all fractions (with oohs and aahs).
The list of all years, as time marches on.
The list of an immortal's goings-on.
List of all words that could ever exist.
It doesn't matter. All are infinite lists.

But all of the numbers from 1 to 2
can never be listed, whatever we do.
No matter what numbers we put on the list
there is one (at the least) that has to be missed.
Yes, although any list will be lively and long
there's always some number that tumbles out wrong.
To find it is simple; it just has to differ
from each number listed, by merely a sliver.
One decimal digit will trigger the trick.
One for each number that lives on that list.
So do face the music and don't be a mystic.
The set of all numbers just cannot be listed.

(Section 4.1: THE STAGGERIN' PYTHAGOREAN)

This poem is a tale of three squares.

Two of them make up a pair.

The third stands alone

(as you've probably known)

'cause there just aren't any spares.

(Section 4.2: The Art Gallery Problem)

There are thieves loose in the museum

(or maybe the coliseum).

But please don't scream.

All we need to see 'em

is v-um divided by 3-um.

With v over 3 guards patrollin'

no painting will ever get stolen.

And you won't find THAT fritter

on Facebook or Twitter

but you might find it by rick-rollin'.

(Section 4.3: Oh no, not Phi Again? -- non-limerick)

"There was a man named Michael Finnegan.

"He had whiskers on his chin again.

"Shaved them off but they grew back in again.

"Poor ol' Michael Finnegan."

Oh no, we're back to feelgood phi-again.

Back to the good ol' jubilee again.

Back to the grand ol' beefy-spree again.

Good ol' feelgood phi-again.

There was a guy named Michael Feegan.

He had a crush on a girl named Reagan.

They met in class over Fibs and phi again.

Good ol' feelgood phi again.

Yep, we're back to the freakin' phi-again.

Fancy as a diamond, rhinestone, or sequin.

Looks like we'll spend this week in phi-again.

Good ol' feelgood phi-again.

It comes up a lot in history-again.

Also nature -- birds and the bees again.

Works of art by daVinci and Klee-again.

Good ol' feelgood phi-again.

(Section 4.4: RUNAWAY CONWAY)

(back to limericks)

Five triangles fit into one

and that's what helps get the job done.

They tile the plane

but it's never the same

no matter how far out we run.

(Section 4.5: HOOKED ON PLATONICS)

O, regular polygons thrive.

It's easy to get them to jive.

But up a dimension --

that leads to dissension

and leaves us with but a mere five.

(Section 4.6: TRENDY BEND-IES)

Who says the shortest is straight?

It can curve like a figure eight.

It can loop de loop

like a hula-hoop

and arrive on the scheduled date.

For instance, a curve on a sphere

will get you from there and to here

or here to there

with time to spare.

You'll arrive by the first of the year.

A bug on the door of the flat

sees a crumb at a spot on the mat.

Does it crawl straight down?

No, it's much too profound

and much too hungry for that.

(Section 4.7: DIMENSIONS FOUR AND BEYOND)

Do you have dimension allergy?

Just try some simple analogy.

Keep up-ping by one.

It's all lots of fun

though it might upset your neurology.

(COURSE SUMMARY -- one more Lewis Carroll)

Marvelous math, so full of quest.

Some say it's cursed, some say it's blest.

It's the start, the end, the whole awesome path.

Math of the evening, marvelous math.

(limerick for the first page of the final)

YOUR NAME! YOUR NAME! YOUR NAME!

Your name is the name of the game.

Please don't forget it

or I can't give you credit

and that would be quite a shame.

(Section 4.8: COURSE SUMMER-Y)

Have a wonderful wonderful summer.

Once it's here, feel free to get dumber.

Forget those riddles

Fermat's Last and Little

and all those primes

and gallery crimes

and the various mods

and the formula, quad.

And forget those pigeons

and long divisions.

Rationals spooky

irrationals kooky.

Those Fibs and phi's

and infinities.

The miles and miles

of Conway tiles.

The cool inventions

of higher dimensions.

You've had enough

of all that great stuff..

Or maybe not.

Still, it's much too hot.

So lower your attention spans

and stand up and dance

to the sound of a different drummer

and have a wonderful summer.

(FAREWELL)

This last is to say our good-byes.

I had lots of fun with you-guys.

I hope our class clowns

did not let you down

and gave you some laughs (and not cries).

CALCULUS LIMERICKS

Diff-ing x-to-the-n is fun.
Change the n to n minus one.
Then go back to n
and use it again
in front of it all, and you're done.

We can diff any power of x
on this whole blasted campus of Drex.
n can be miniscule
as an H-molecule
or as large as Tyranno Rex.

It is not my intention to vex.
It is not my intention to hex.
My only intention
is merely to mention:
The diff of x-square is 2x.

It is not my intention to glare.
It is not my intention to scare.
My only intention
is merely to mention:
The diff of x-cube's 3x-square.

There's a general rule for all this
(in case someone pulls a pop quiz):
n steps down two ways
for the rest of its days
and x remains right where it is.

"Than receive it is better to give."
Also, "live, live, and let live."
These are words to clutch
but not as much
as "velocity equals deriv".

(General Power Rule)
Are you bored with powers of x?
What would you like to do next?
Some powers of g
will do nicely
but watch out for the special effects.

On, Dasher! On, Dancer! On, Prancer!
Don't forget the power in the answer.
It gives it might
and makes it right
also a little fancier.

For lazy execs and high techs
presenting: e to the x.
To diff it's a cinch.
To the nearest square-inch
just use your do-nothing reflex.

Three cheers for parentheses!
They make forests out of trees.
But be careful about
what's in and what's out
or they'll look like they have some disease.

(Product Rule)
Don't forget -- f appears twice.
g will behave likewise.
If you make like a dunce
and write them just once
... well, you'll get what you get for half-price.

Said a wiseguy named Georgy O'Porgy,
"Let's have a Quotient Rule orgy.
"On top, to be nice
"let g appear twice
"and then on the bottom one more g."

(Chain Rule)
Rinky, dinky, dinky.
g provides the link-y.
But the x and the f
are not to be left
out, unless we run out of ink-y.

Don't forget -- g appears twice.
I think that advice is quite nice.
And f appears once
these early spring months
and summer and winter likewise.

Careful now, don't lose g-prime.
You could write it first every time.
And what to write second?
Well, there's f to be reckoned
before we can finish this rhyme.

(Double Chain Rule -- functions of functions of functions)
g provides the link-y
but so does h, the stinky.
If another meanie
appears on the scene-y
we WILL run out of ink-y.

Once again, g appears twice.
But h, little pest, appears thrice.
But hey! it's a party
healthy and hearty
and h needs to break the ice.

And as long as it's party time
let's not forget ol' prime.
Yep, each guest gets diff'd
simple and swift
except for x, the slime.

Ln x, you can't just be.
You have to get diff'd, yessirree.
So give ln the slip
then take the recip
to get one-over-x, easily.

(General Ln Rule)
On bottom goes the copy.
And what goes on the top-py?
Why, the diff, g-prime.
It makes it rhyme.
And please don't write it sloppy.

(Implicit Diff.)
We're so used to y on one side
and we'd diff it with joy and with pride.
But in this crazy case
y's all over the place
and x goes along for the ride.

So we've x/s and y's galore.
How can we tackle this chore?
Why, we diff regardless.
Perhaps it seems heartless
but mindless it isn't, for sure.

Now, as we proceed down the river
and diff those terms, sliver by sliver
remember, the Chain Rule
will be the main rule
but Product Rule isn't chopped liver.

(l'Hopital's Rule -- Good Golly, Miss Milly / here comes l'Hopital'y.))
Don't forget -- hafta diff twice.
It adds spice to this slice of life.
Two is more fun
than just plain ol' one
and ten-million times more precise.

There was a fair maid from Nepal
who was expert in ol' l'Hospital.
She diff'd 'til she dropped
on bottom and top
then murmured "Nice knowing y'all."

Said a brilliant young co-ed named Ricki
"Watch out, though, this might be a tricky.
"The quot. of limits
"could be purely legits
"and we won't have to diff a whick-y."

A strapping young math whiz named Sammy
when presented a double-whammy
said "Wow! Now I'll get
"to diff four times, I bet."
With that spirit, he'll soon earn a Grammy.

(Exponentials dominate powers -- comparing b^x with x^p, as x goes to infinity)
No matter how little is b
and no matter how big is p.
b to the x
is the one that out-treks
at least eventually.

It's not my intention to confuse.
It's not my intention to bruise.
My only intention
is merely to mention:
Anti-diff x, get two 2's.

It is not my intention to tease.
It is not my intention to tweeze.
My only intention
is merely to mention:
Anti-diff x-square, get two 3's.

It is not my intention to force.
It is not my intention to coerce.
My only intention
is merely to mention:
Anti-diff x-cube, get two 4's.

A jingle:
We need to be fussy
about that + C.

(Power Rule of Anti-Diff-ing)
There's a general rule for all this
(and it's something you don't want to miss):
n moves up and down
all over the town
and x remains right where it is.

Minus-one is a cool special case
delicious and dainty as lace.
So don't play the hero.
Don't divide by zero.
If you do, be sure to erase.

(Integrating Exponentials)
And now here's a grave admonition
delivered with proper precision:
It's about that k.
It steps down just ONE way
and the x doesn't go where it isn't.

(Curve-sketching)
We can plot and plot 'til we plotz.
But we've got to plot the right spots.
Or those lows and those highs

could elude us like flies.
Likewise the flips and the flops.

(Anti-diff-ing sin's and cos's)
A lean lazy lad, name of Jackson
is always forgetting that fraction.
Indeed, he should put
that k underfoot
but he's much too busy relaxin'.

There was a young man named Kareem
who explained, "For a local extreme
"the tangent at a
"to rest must lay
"and we wish it the pleasant-est dream."

Said his kissin' cousin Trix
"But not vice versa -- nix.
"That tangent can flatten
"as low as Manhattan
"with no min, no max, just a mix."

A fair maid from North Minnesota
was drawing a steep asymptot-a.
When it got 'way too high
she murmured bye-bye
and mourned not a single iota.

(Integration by Parts)
"We need f and g-prime," said Mitch
"and it matters which is which."
"But not to worry,"
said his cousin Murray.
"It doesn't work out, we'll switch."

(First Fundamental Theorem of Calculus)
Don't forget -- evaluate twice.
Sorry but once won't suffice.
Howe'er, the subtraction
is a single-action
and I would say that's very nice.

(initial value problems)
If we're given a function's deriv
the function itself we can give
but to only within

+ C, what a sin.
We need one more fact to work with

(Second Fundamental Theorem of Calculus)
Take f, whatever it be
and make it a function of t.
Then integrate (def)
that poor little f
a to x (not quite a to z).

Of course, if we diff those effects
we'll get back that f -- it connects
yup, just plain ol' f
as 'twould be our pref
and it's back as a function of x.

Root-a-toot toot-a-falutin'.
It's time for some substitutin'.
Take stuff on display
and collapse it away
right along with Leibniz and Newton.

The differential
is essential.

(Approximate definite integration)
The trapezoid rule can be fun.
All those 2's will get the job done.
But watch out, my friends
for the left and right ends.
At those we will only need 1.

(and one about Simpson's)
To say it in so many words:
this thing is a matter of thirds.
So nail that ol' Simp
and don't be a wimp.
Dividing by 2 is for nerds.

(Area between Curves)
If the graph of x-to-the-sixth
with x-to-the-eighth is, not mixed
only placed, for our practice
on the same set of axes
then subtract to find what's in betwixt.

Don't forget, curves just might cross
in which case you should take time to pause
so you don't subtract
when add's where it's at.
'Twould be a lamentable loss.

To split or not to split?
'Til the question that stymied Hamlet.
But when you're in doubt
don't leave the split out
though it's harder, I admit.

We could sit all prim and proper
privileged as gold and copper
but 'twould perk our careers
if we'd get off our rears
and become intersection-hoppers.

(Solids of Revolution)
A strapping young woman named Evvie
was handed a solid of rev-y
and asked for the volume.
She answered, quite solemn
"it's not very big but it's heavy."

(how to find the volume of a solid of revolution-- Step 1)
Hear ye, gentlemen and gentle ladies:
The key is to first find the radius
as a function of y
or of x -- come, let's try
in this classroom of Arcadia's.

(Steps 2, 3, and 4)
Now we've got r(x) with great flare
so it's high-time to figure its square
and then multiply
by good ol' pi
integrate and we're done -- so there!

(Average Value of a Function)
Riki tiki tavi.
Here's some calculus savvy.
The inteGRAL
o'er the interVAL
will give us our function's av-y.

(Integration Techniques: Sine-Saving)
Said a technique freak named Zeek
“If you think sine-saving is chic
“and you want to save
“yourself into the grave
“try cos and tan and sec.”

Here’s a little ditty
helpful if not cute:
What we save is the deriv
of what we substitute.

(Deriv’s of Trig Functions)
A darling named Clementine
said, “First sine, then cos, then sine.
“And the minus and plus
“make things even wuss.
“Can’t it make up its mind?”

A Non-Limerick:
Can we integrate tan?
Yes, we can.
Can we integrate secant?
No, we can’t.

(Trig-triggers -- i.e., trigonometric substitutions)
A trig-trigging trickster from Beacon
is stuck on an odd-powered secant.
An integral table
would render him able
but his conscious is prodding “no peekin’ “.

Trig, trigger, triggerest.
Try not to be too vigorous.
If you trig too much trig
it’ll get too big
and you’ll fall down just like Icarus.

(Hyperbolic Functions) (Are you a hyperbolic-aholic?)
Take the trig I.D.’s, all kinds
add h to those cos’s and sin’s
and this derring-do
leaves them all still true
except for a few miinus-signs.

(Completing the Square)
When the middle term nerdily lurks
use this to get rid of the jerk.
Sometimes a wild guess
will get rid of the mess.
And algebra always works.

(Approximate Integration)
The Trapezoid Rule can be fun.
All those 2's will get the job done.
But watch out, my friends
for the left and right ends.
At those we will need only 1.

Cheer up. (Don't look like such wrecks.)
I know this seems pretty complex.
And there's many a y
where the midponts lie
but there's only one delta-x.

Just look at those sixes and fours!
They're the prettiest things in this course.
And this cool rule of Simpson
will stay sweet and winsome
once you've become sophomores.

(Improper Integrals)
Said a ship-shape chap from the Congo
"Since our region is infinite long-o
"it's likely that you'd
"be inclined to conclude
"that it's infinite big, but you're wrong-o."

Said a dashing young bloke named Apollo
"Since our region is infinite tall-o
"It's likely you'll dig
"it be infinite big
"but sorry, that doesn't quite follow."

(Separable Differential Equations)
A ditzo from hither or thither
refused to get all in a tither.
"'Tis true," said he, "I
"can't tell x from y
"but can't tell left from right-y, either."

The vertical line test's a pity.
It stops curves from being pretty.
But we can make loops
without saying "Ooops?"
with parametric graph-iti.

(Polar Coordinates)
With these we can also be arty.
We can have a plotting party.
While away the hours
making petals and flowers
along with Mercenne and Descartes.

Would you like to come up to the board
and draw r and theta coord's?
We'll get hulas and hoops
and loop de loops
and other delicious rewards.

(Sum of an infinite geometric series)
Let a be the first to assert
Let r be the ratio pert
put a on the top
Let r from 1 drop
-- don't worry, it won't get hurt.

(Infinite Series -- Comparison Test)
Smaller than small is small.
Taller than tall is tall.
And that's how it goes
and that's how one knows
whether anything happens at all.

(Integral Test)
If we know what happens with n's
all x's will follow, my friends.
It's all essentially
the same, consequentially
and on each the other depends.

(Alternating Series)
However they rageth and roareth
and wobbleth back and forth,
you'll eventually find
that they make up their mind

someplace between south and north.

(MacClaurin Series for a given function))
The first in our fine repertoire
is one over one minus r.
We mustn't forget it
for where we are headed
or else we won't head very far.

An industrious lad from the near East
was summing a long Taylor series.
At the twentieth head
he just shrugged and said,
"That's as far as I'm going, my dearies."

(The formula for the coefficient of x^n in the power series for f(x))
To find it is easy as pie
(especially if you try).
There's an exclamation
and a derivation
one low, the other high.

(Applications of the definite integral)
Applications come in immensity.
One of them involves density.
Another, alas
is center of mass
which feels with us the intensity.

(Partial Deriv's)
We x 'em and we y 'em.
But not both at the same time.
f_x is one
f_y's also fun
though we never get to f-prime.

(End-of-Term Farewell)
Yes, I know, it's 10:49.
But this proof needs one more line.
I promise I'll write
with all of my might.
(Keep mum if I'm off by a sign.)

COMPLEX ANALYSIS

A reality complex had Fred.
He felt kind of bad in the head.
So he took Complex Var
with Cohen Mar
got a complex complex instead.

There was a young lady named Suzie
who couldn't add 2z and 2z.
She said, "Can't it be
"without that ol' z?"
She's being a little too choosy.

(nth Roots of Unity)
There are n ot them sprawled on a wheel.
Among them at most two are real.
The others must go
half above, half below.
But they get paired off in the deal.

e-to-the-z is exotic.
It changes a sum to a product.
It has no root
but it's kind of cute
and it's vertically periodic.

A young man named Kenny macKenzie
had trouble computing ln z.
He breezed through the r-part
with the slickness of pop art
but the theta-part gave him a frenzy.

Here's the key to z-to-the-c:
It's just e-to-the-c-ln-z.
But watch for ln-y.
Its values are many
though sometimes e makes them agree.

(Said Sin and Cos)
"In trig and in calc, all through
one was the limit we knew.
But now we're set free.

Now we can be
anything we desire such as two."

How smart the conditions of C-R !
They're almost as brilliant as we are.
But they're known for their sly-ness.
Watch out for that minus
or we'll spend half the night in the E.R.

Yes, here they come, hot off the griddle
spiffy and spicy and little.
One sports a plus
the other mi-nus
and none of them sports the middle.

The research team Cauchy and Goursat
was busting its brain and its torso.
They went huffin' and puffin'
but came up with nothin'.
Now they're living on noodles and orzo.

The pretty professor from Jersey
just stood there and pleaded for mercy.
She said "in complex
"v-y equals u-x --
"or maybe it's vice vers-y."

i after e after u
after pi after n after 2
after p'renthesis ln
and then no more spellin'
unless we can find something new.
(Thanks to Bob Blackard for getting me started on this one.)

Our pretty professor's a rarity.
Her lectures the height of clarity.
Except that when
she does minus-one to the n
n turns out to be the wrong parity.

The pretty professor, our rarity
was engulfed in hyper-hilarity
'cause again she had spun
a spare minus-one.
She said, "Well, we can give it to charity."

(About $\oint f(x)/(z-z_o)^n$)
When C's end is at its beginnin'
ask "Is z-nought out'n or in'n ?"
Draw the point, draw the curve.
Then stand back and observe.
If in doubt get a second opin'on.

Computing it's easy as pie
(especially if you try).
There's a derivation
and an exclamation
and don't forget two pi i.

This classroom is water- and sun-proof
and poison- and noisin'- and gun-proof.
It's as pure as the Gospel
of everything poss'ble
except that it's not minus-one-proof.

Geometric ser's are a gem.
But let's add that little m --
OUTSIDE, if you please
the parentheses.
Did you hope I'd forget about them?

The last of those spunky Mohicans
was at work on a long Cauchy sequence.
It was i - pi - y
from Mon to Fri
and he took it home on the weekends.

Can you count? Then here's one for you.
One less z to the minus-two.
Write 1, 2, 3
then blank, z, z
and the powers and the plus-signs, too.

A fair maid from Northern Miami
was at work on the triple-whammy.
She tried subtraction
and partial fractions.
Then she called for her pappy and mammy.

A lazy young lad named Laurent-o
used his series to integrate pronto.
Just b-one will do
and as for b-two
you need only find that it you want to.

O what will that fair maid named Tessa do?
She's been after a simple-pole residue.
She says, "NOW I see.
"It's q-prime UNDER p.
"Then I plug in z-nought -- or I guess I do."

TEACH-TONE PHONE (Remember, I gave out my phone number? Well, no one ever called, but if you had, here's what my recording said -- See what you missed?)
For Cauchy-Goursat
press O.
For some integral fun
press 1.
For a residue
press 2.
For I / (I - z)
press 3.
For contours galore
press 4.
But to find out the fate of a minus sign
be patient, please, and stay on the line.

LINEAR ALGEBRA

Allow me some words of great wisdom
about any linear system:
It is represented
by a matrix, augmented
which explains this course's existence.

Do row op upon row op.
Keep doing them 'til you drop
or until rectangular
becomes triangular.
(And the triangle's on top.)

The form which is just-plain row ech-
means surely you'll still have to thresh
with back substitution
so keep on reducin'
to reduced row ech in the flesh.

(Matrix multiplication)
Just pair off each row with each column.
Take the sum of those products so solemn.
Yes, beat the odds
with those scalar prods
or whatever the devil ya call 'em.

(Definition of Determinant)
O, nothing could ever be finer
than replacing A major with minor.
Of course, for most n
we must do it again.
And the signs will get sign-er and sign-er.

(det A x det B = det AB)
I don't mean to cause you to squirm
but the computations confirm
that determ before prod
(in the eyes of God)
gives the prod before the determ.

When we wish that our fingers were toes
and we wish that the columns were rows
and the i's and the j's
are turned the wrong ways,
'tis time to bring on the transpose.

(how to find A-inverse)
On cofactors get yourself versed.
But start with the transpose first.
And do not forget
to divide by det
and you'll wind up forever cursed.

If you know how to cut and to paste
and can figure determ's with great haste
and one more aside:
if you've learned to divide
then by Cramer's cool rule you are graced.

(to decide whether a given set of functions is linearly independent)
Line up all the functions in sight.
Then diff them with all of your might.
Then sing a sweet song
of right and of wrong
as we get that ol' Wronskian right.

A young man named Timothy Tigen
was searching for values eigen.
But the characteristic
and other logistics
were such that he needed to try 'gain.

(how to diagonalize)
E-values will get us big-D.
E-vectors will get us homefree.
Yep, string up the specs
and line up the x
and we'll get where we needed to be.

If its set of e-vectors is sizeable
then A is diagonalizable.
And the converse, too
has been tried and true.
(I hope that is all recognizable.)

(how to exponentiate a matrix)
Proceed as the previous verse
has instructed (for better or worse).
Treat D as you're tempted
and X is exempted
except we'll still need its inverse.

(Definition of vector space)
We plus 'em and muss 'em and less 'em.
We shrink 'em and stretch 'em and press 'em.
And still they will be
inside our big-V.
Such fine loyal souls, the Lord bless 'em.

(subspaces)
We can push and pull and stress
and otherwise make a big mess
and they'll not only be
inside our big-V
they'll also remain in big-S.

Can we get infinity from fin
(without committing a sin)?
Oh yes we sure can
with the kind help of span.
(After all, it's the past tense of spin.)

(dimension)
Each basis goes by its own name.
Each vector goes by its own aim.
But if you would count
the number-amount
the answers will turn out the same.

Base B can be used to express
any member of V or of S.
But the thigamabob
that will pull off the job
depend on which B we access.

To get thigamabob for C
from thigamabob for B
will require the rendition
of matrix, transition
and seldom will they agree.

(linear operators)
They're linear, down to the core.
They take zero to zero, no more.
And sums and doubles
and all other troubles
to what they had been before.

(Multiplication by any matrix is a linear operator.)
Said a fair young maid named Alexis
"Ax will double if x does
and A takes a sum
into something hum-drum
and there won't be any more extras."

(rank)
A matrix has n column-vectors.
Indeed, it's a vector collector.
It's all touchy-feel-y
but how many, really
are there and how many are extras?

(Column rank = row rank)
You can put them in lines or in layers.
And then, if you say the right prayers
it won't matter which.
The quarter-turn switch
won't change what's essentially there.

(Every linear operator = left multiplication by some matrix.)
Just take what L does to each e
and line them all up merrily.
And that's a good way
to get your big-A.
(Watch out so you don't get a B.)

Let's all do a little Gram-Schmidt
get orthogonal vectors that fit.
At every stage k
I'll show you the way.
(At k = n we can quit.)

VECTOR ANALYSIS

Here's something we might contemplate:
A vector's an arrow so straight.
For forces, its length
expresses its strength
and direction expresses its fate.

Two vectors, whatever the size
can be added, component-wize
so make so this rhymes
by adding three times
then put in the k, j, and i's.

(Scalar Multiplicatoin)
A spunky young woman named Ralpha
would multiply vectors by alpha.
She thought it quite nice
to multiply thrice
in this city of Philadal-pha.

(Scalar Product)
Here's the most fun you ever had:
Take products three times and then add.
Yes, do mind your mommas
and DON'T put in commas
(or momma will tell your Dad).

(Vector Product)
In this glossy and glorious firmament
nothing's more perfect or permanent
or more fancy-free
for u criss-cross v
than our ever-lovin' determinant.

i, j, and k are cute
and they're specially fixed to compute.
E.g., i cross j
is lower-case k.
But as arrows, they're too short to shoot.

(Some curve parametrization hints)
Straight lines are quite matter-of-fact
if you know how to add and subtract.
True, there's multiplication

in this situation
but nothing on which you need act.

Although we are all mere amateurs
in this business of making parameters
when it's circles in question
I have a suggestion:
Try not to forget the diameters.

(Formula for Curvature)
r-dot dot r-dot-dot.
Yes, sometimes we dot a lot.
But dot-dot-dot?
Or r-quad-dot?
. . . Well, sometimes we do not.

For that curvature for t' compute
that formula's really a beaut.
But when figuring kappa
do mind your Pappa
and don't you forget that square root.

It's easy and fun to do grad.
It makes us feel gleesome and glad.
Don't you feel so alive
when you partial-deriv?
But commas, now -- don't you dare add.

Here's something no one will object to:
When doing the div please expect to
takes partials, three
of componenets of v
and be careful which with respect to.

Aloha, tres bien, and shalom.
May I mention a vital syndrome?
Towards the end of the div
you should get additive
(or just wait 'til your father gets home).

Now, the hardest is curl, we agree.
But let's have a determinant spree.
First, three vectors little
next, del in the middle
and then on the bottom goes v.

All told (and telling it true)
there's more in the curl to do.
We've got partials six
and note how they mix
and the answer's a vector, too.

(Directional Deriv.)
If we're out in the wide blue yon
rates of change depend, which directi-on.
So divide b by mag
then dot it with grad
and we'll find out how quickly we've gone.

(Unit Normals to Surfaces Given in the Form f(x, y, z) = C)
Said a dashing young man named Aeneas
"I've got a few dandy ideas:
"Just take our f's grad
"divide by the mag
"and it doesn't much matter what C is."

(Reality check)
When these poems you so dutif'ly edit
keep one thing in mind (to my credit):
I realize that mag
does not rhyme with grad
but the point is: YOU won't forget it.

(Surface Parametrization Hints -- Plane)
If you want to be this term's winner
remember, the keyword is linear.
So let us let z be
ua + vb
(for integrals outer and inner).

(Cylinder)
First, think of the xy-plane.
(That's not such a drain on the brain.)
Next, move once around
and then up and down.
If tired out, go hop on the train.

(Sphere)
Said a student from South Singapore
"We use cosines and sines galore

“or the sum of three squares
“would give us nightmares --
“Yes, that’s what the trig stuff is for.”

(Line Integrals)
And now we’ve got funct-i-ons three.
What could the integral be?
The answer, essential:
Component, tangential.
It gets things in terms of small-t.

(Surface Integrals)
Next, functions (three) with a surface
(And Murphy’s Law gets more Murphish).
But the answer, informal:
Component, normal
to get u’s and v’s at our service.

(Triple Integrals)
Now, triple int’s can be quite chic
so of them we’ll now start to speak.
But one thing clear:
No vectors here
(or you’re grounded for a week).

(Divergence Theorem)
Next, the surface int. above
that we’ve all come to know and love
will be giving ‘way
on this fabulous day
to a triple int. of the div. of.

(Stokes Theorem)
And now it is late, not early.
‘Tis time for Stokes’ Theorem, surely.
Any int. (closed line)
can be redesigned
as an int. involving the curl-y.

(Reality Check)
Again, as these lim’ricks you edit
remember one thing (to my credit):
I know that designed
does not rhyme with line
but it rhymes more than how Stokes said it.

DIFF EQ

A fair maid from Northwest Virginia
was solving a first-order linear.
But that exponential int
was too large to print
so she just made it shorter and skinnier.

A starry-eyed lad from Wisconsin
was hoping those coeff's would be constant.
But his hopes turned to hexes
at all of those x's.
He shrugged and remarked "Stuff and nonsense".

There was a young woman named Sally.
Legendre was right up her alley.
"Minus-x-square plus one
"is what starts the fun
"and n-square plus n's the finale."

Her cousin named Mary Magee
said "Bessel seems besser to me.
"x-square WITHOUT one
"seems much more fun
"and nu's cooler than n," said she.

Their cousin from Doodle-Skadiddle
said, "Yes, but what's in the middle?"
Answered they, "we suspects
"it's x or 2x.
"We're feeling a big non-committal."

An attractive young ms. named Cassandra
was solving equations Legendre.
a-nought was do-able
a-one construable
but a-(n+2) was beyond 'er.

There was a fine fellow named Ian
who was ready to give up on p_n.
He said, "Too much mess
"with the n and the x".

And sometimes I can't help agreein'.

Let's do J-n versus P-n.
The former's a much smaller bein'
with much less subtraction
and all told, less action
'cept: more plus-signs in between.

We've y-one but not y-two.
What're we gonna do?
Just look to the board
at Red. of Ord.
how, instead of C, we try u.

We've y-one and y-two sublime.
But r 's not zero this time.
So do Var. of Par.
with Cohen Mar
then stand up and drink to l'Hyam.

(Picard's Successive Approximations)
Again and again and again
from n-1 to n.
We get closer and closer
(If it's cos, we get cos-er)
but making no promises when.

(Euler-Cauchy Equations)
They sport both dx and dy
and an x-square and a by.
And the thing to do next:
guess a power of x.
When you plug it in, you'll soon see why.

(Bessel Functions)
In these guys the x appears twice
'cause the powers get strategically sliced.
The x that is mighty
goes to the right-y.
To the left goes the x that is nice.

PDE

A lad thought he'd service society
by studying math and psychiatry.
But his plans went a-ragin'
with the first wave equation
'cause he got Separation Anxiety.

Four, Fourier, Fouriest --
Isn't it just-plain glorious?
How, from sin's and cos's
we get (by osmosis)
so much, and emerge victorious.

(Fourier Series in Exponential Form)
Those functions e-to-the-in-x
can be treated like cos x and sin x.
And if they can't
call me Bell Atlant-
or try MCI or Ninex.

(Fourier Integral)
Are we feeling all morbid and mopey
just 'cause f(x) has no p?
Don't be so timid.
Go to the limit.
When there's life, there's always hope-y.

It's Jan. 14 and our prof
by a factor of TWO is off.
Is she going to cry?
Says she, "No, not I.
"In fact, I'm more likely to laugh."

Do you think this is getting hum-drum?
It's time to consider a drum-drum.
We'll get double series
and double theories
and B's and B*'s under 'em.

Next we make our rod very lengthy
thus increasing its impact and strength-y.
So we've lost the L

but gained integrell.
For thy patience, I humbly thank thee.

There was a spry guy from North Saigon
who was searching for values eigen-.
But the boundary conditions
surpassed all his wishin's
and wouldn't let bygones be bygone.

Oh, trig functions bellow and bark
and Bessel's go bump in the dark.
but Sturm-Liouville creatures
have all sorts of features
and fill up the whole Noah's ark.

ABSTRACT ALGEBRA

We're a bunch of nincompoops.
We can't get a grip on our groups.
We ask, in a frenzy
"Z_n or nZ?"
Guess wrong, so grin and go "Ooops".

This quarter we're all pretty group-y.
It can get pretty meaty and soup-y.
Z_3 order 3
Z_p order p
and D-sub-p order 2p.

(You know how that religious mathematician Kronecker said "God created the integers; the rest is the work of man" -- Well . . .)
"Integers are neat," said God
"both the even and the odd.
"So I'll give you some, Kronecker
"for Christmas or Chanukkah.
"Do you mind if I package them mod?"

Said a group theory pro named McClellan
"We've too many isom and elem
"and homom and autom
"but already bought 'em.
"Perhaps we could try to re-sell 'em."

(Automorphisms)
Some're outer and some're inner.
All're autom's (although it's winter).
Some turn out to be
the mere identity.
But that's good when you're a beginner.

Rub-a-dub, rub-a-dub, bub-bub.
What, pray tell, what, is this hub-bub?
It's subgroups so cool.
They obey the strict rule:
Order-wise, they divide what they're sub ub.

In the kitchen you'll hear lots of slubbering.
In the bathroom you might see a tub ring.

But the classroom, I claim
is quite tidy and tame.
All it's got is a subgroup or subring.

(More dialog between God and Kronecker)
"Integers are great," said God
"and it's fun doing sum and prod.
"But I've now got some notions
"to tackle the quotients"
and all except K oo'd and ah'd.

"It was nice dividing by bd.
"But now I'm getting more greedy."
Thus spake our Lord
standing straight at the board
writing Q-bracket-x and Q.E.D.

"One was good for a laugh.
"And two was a treat for my staff.
"But I've now got a hankerin'
"to do some tankerin'.
"I'll start with one-and-a-half."

"For seven long days labored I
"with integers low, then high.
"But now 'tis day eight.
"'Tis time to create
"quotients and roots and pi."

Deep in waters hot, not tepid
feeling rueful but intrepid
our two creators
and integer-traitors
shrugged "Sorry, we just can't he'p it."

"Please forgive me," said Kronecker L.
"I hope you won't send me to hell.
"But x caught my attention
"and x led to extension"
and the rest we know only too well.

Yes, pity the genius and hero
who fiddled, but not like Nero.
He started with F.
Now he's got nothing left
for he sold his soul for a zero.

(Primitive Element Theorem)
A fair maid from Alabam-y
was given a double whammy.
But she knew how to mingle
so's to make it a single
and ace the final exam-y.

(When is F(a) isomorphic to F(x) ?)
If about a we have no spec's
then a might as well be x.
And e and pi
are good a's to try
though it's not quite clear how it checks.

We might grow up to be tax-collectors
or city or country inspectors.
But Chap. 23
ensures that we
will never be angle-trisectors.

(God confesses to Kronecker)
First I couldn't stop at ten.
The I couldn't stop at n.
Then Z and then Q
and the square root of two.
But I've now come to C. Amen.

(Epilogue: by God and Kronecker)
We work together like brothers.
One creates, the other discovers.
And Galleon writes.
And Cohen recites.
And we welcome any others.

PROBABILITY

(Set theory)
Hippity hippity hoppity.
A set can be built through a property
with nouns and verbs
and blubs and blurbs
or any ol' thigamabob-ity.

A set can also be seen
as an element-making machine.
That set will consist
of all s in some list
with commas in between.

A fair maid from North Beelzabub said
"I don't want to get us all upset
"but if s in B
"implies s in C
"then B is of C a subset."

(The empty set)
Here comes phi, consisting of nada
and neither your mada or fada
can fill-er it up
for lunch or for sup
not even with air or with wada.

(Union and Intersection)
The union of 9th and Race
is something we'd care not pace.
But the intersection
leads to detection
of that fabulous Thai lunchplace.

(Universal Set)
Here is one more thigamajig
that we'll need for this whole shindig.
Yup, time for big-U
too big to be true
but not too true to be big.

(Complement)
Whatever set A hath not
is what A-prime hath got.
And their intersection
defies detection
but their union is a lot.

(Cartesian Product)
Let A be of women a set
and B be of men; then we get
from A-cross-B troubles
all possible couples
providing we know they're all het.

($n(A \cup B) = n(A) + n(B)$)
Here's one more essential point:
If A is from B disjoint
then to give you, I'm glad
permission to add
at the risk of seeming flamboyant.

($n(A \cup B) = n(A) + n(B) - n(A \cap B)$)
And now here's another cool fact
on which you might soon need to act:
You still, my comrade
have permission to add
and orders to also subtract.

(to calculate $P_{n,r}$)
Write down n, and then when you're done
write parentheses n - 1.
The next thing to do:
Write n - 2.
Keep going, but not down to none.

(to calculate $C_{n,r}$)
Here's something that's redder than henna:
our blood when computing $C_{n,r}$
and the sweat and the tears
as we and our peers
do canceling more than we wanna.

A fair maid named Mary Maltese
got her P's all mixed up with her C's.

She said “I do figure
“the P’s must be bigger
“but hey! there are no guarantees.”

Said her kissin’ cousin Muriel
“I’ll give you a little tutorial.
“In case you’ve forgotten
“the C’s, on the bottom,
“sport an extra r-factorial.”

(Mississippi & Co.)
And now here’s a new little game:
permuting when some are the same.
Permuting these some
will feel sort of numb
although there might still be some pain.

(Reality Check)
As these poems you so dutifully edit
keep one thing in mind, to my credit:
I realize that pain
does not rhyme with same.
But it will if you’re less anal-headed.

(Binomial Formula)
I’m sure you’ve a loyal FOIL-er
and I won’t be a FOIL-er spoiler.
But when n, for its britch
gets too big, you should switch
and be to this formula loyaller.

(Outcomes)
When you do an experiment-y
the possible outcomes are plenty.
And it’s kind of fun
to find out which one.
(It’s a good one, Deo volente.)

(Sample Space)
And now let us go with the flow.
Take the set of all outcomes, just so.
Yep, time for big-S.
Yes! yes! yes!
Don’t you dare say no!, no!, no!

(Events)
Here's something we'll use infinitum:
Events (We'll both say 'em and write 'em.)
And to poet-ize:
Size-wize, they lies
between the two previous items.

(Another Reality Check)
As these poems you all superintend
allow me to make some amends.
I know that "they lies"
is bad grammar, you-guys.
But what's one s among friends?

(Probability at last)
We can now introduce big-P.
It means probability.
P of S is one.
P of phi is none.
In between lies P of E.

(Probability of Disjoint Unions)
Said a dude with a shrewd attitude
"If events doth each other preclude
"then it's easy to tune in
"the prob of their union
"providing we're in the right mood."

($P(E \cup F) = P(E) + P(F) - P(E \cap F)$)
The above should inspire deja vu.
Take the rule from 6.2
change the n to P.
It seems clear to me.
I hope it seems clear to you.

($P(E') = 1 - P(E)$)
For the prob of E's compliment
take the prob of that E-event
subtract it from one
and you'll be all done.
(You can give it up for Lent.)

If the prob of each outcome you do know
then by Jupiter!, Jove! and by Juno!
the sum of them all
will happen to fall

extremely close to uno.

(P(E) = n(E) / n(S))
And now here's our big opportune
to do what we've always been doin' --
permute and combine
but twice this time
on this beautiful late-afternoon.

(One More Reality Check)
As these poems you so dutif'ly edit
keep one thing in mind, to my credit:
I know that time
and combine don't rhyme
but I'm not overly poetic.

(Expected Value)
First, the possible values collect.
And then, to their prob's connect.
The grand finale:
Expected val.
What else could we hope to expect?

(Variance and Standard Deviation)
Here's a question a trifle mischevious:
Are the deviations devious?
The answer, I'm 'fraid
can't be bought at Rite Aid.
Perhaps we can get it at CVS.

(Binomial Distribution)
n trials, each a yes or a no.
We want x yes's, just so.
The prob of that mix
involves n-choose-x
and the p's and the q's also.

There's no need to moan and groan
'cause it's easy to do Poisson.
There's nary a care
with Cohen Mar
and less with Marion Cohen.

(Markov chains)
And now let us make a mad dash
-- a transition matrices bash
with gobs and gobs
of “throwing prob’s”
regardless of whether we catch.

To see how to get here from thereabout
P is all that we care about.
But we’re gonna need v
most definitely
for info concerning the whereabout.

(The Last Reality Check, promise...)
As these verses you classify
as to whether the rhymes apply
you’ll notice that mix
does not rhyme with x.
The same would be true of y.

(Some farewell wisdom)
So now that we’ve had quite our fill
here’s a life-lesson from my quill:
The prob’s, woebetide
won’t be on our side
but the possibilities will.

STAT

(Intros)
I promise, the mean isn't mean.
Nor the deviation obscene.
Nor is any statistic
overtly sadistic.
It hurts, I'll give you morphine.

"The trouble with a kitten is that
"eventually is becomes a cat."
And the trouble with Probability is that
eventually it becomes Stat.

(Summary)
Too big is a populace
to get all the info in place.
So we gather a sample
(one that is ample)
and infer 'til we're blue in the face.

(Data-Values)
You write them all down on a sheet.
(You try hard to make them look neat.)
Some will be qual-
and some will be quant-
and ne'er the twain shall meet.

(Reality Check)
As that last you so dutif'ly edit
keep one thing in mind to my credit:
I know qual- and quant-
(although I might want)
don't rhyme -- I'm still glad I said it.

Continuous, maybe discrete
in inches, in yards, or in feet
they're numbers all
some big, some small
and some of them dare to repeat.

(for a Population)
The size is denoted big-N.

The mean is small-mu, and so then
for standard dev, sig-
ma (little, not big)
or so we would recommend.

(for a Sample)
Small-n, for the much smaller size
(How could we proceed otherwise?).
Then take the wild guess
of X-bar and small-s
or so we would strongly advise.

(Histograms)
The small x's, if you please
are the values that big-X doth seize
and the various y
so high and so spry
are the relative frequencies.

(z-scores -- "x minus mu / and Tyler, too")
Three cheers for x minus mu.
It brings on those z-scores for you.
For trains or for cats
it makes for good Stats.
But there's stuff on the bottom, too.

(Mean = average)
Just add up the numbers, each one
divide by the n ad you're done.
Gee, that sure was quick. (There must be some trici.)
Time sure flies when you're having fun.

(Standard deviation)
Each number's some distance from mu
and those each have squares (powers 2).
Now, add those squares duly
divide by n truly
then take the square root and you're through.

(The var is the square. -- a non-limerick)
This next is something you might love
since it is almost like above
Do everything the very same
except that last -- no, no! for shame!

Take no square root -- no, no, no, no!
Just leave it as it is, just so.
I know it's tempting, but refrain
from that last step, control your brain.
I know it's hard, I know you'll grapple.
But don't be like Eve, don't eat that apple.

(Another standard deviation non-limerick, based on a popular song)
"There was a man named Michael Finnegan.
"He had whiskers on his chin-agin."
"He said 'Oh no, must I divide by n again.'
"Yes, you must, so begin again."
Epilogue: He was quite tired of that shenanigan.
But he began again.

(A non-limerick)
The less-then probs are truly a breeze
for you and your TI83's.
And the more-than probs, I'm happy to say
are a mere subtract-from-one step away.
And last but not least are the probs between.
TWO steps away and pretty routine.
So get it straight forevermore
and you'll do just fine in Chapter 4.

Three cheers for the theorem of Bayes!
We'll cheer for the rest of our days.
We'll get P-D-T
and P-T-D
as we shout out our hip hp hoorays.

The theorem of Bayes is fun.
And it's certain to get the job done
with sensitivity
and specificity
and some help from P-D and one.

(Binomial distributions)
There's one for each n and each p
in this functional big family.
n, for our files
is the number of trials
and p is the rel-frequency.

(to find P(X(n,p) = x) on the TI83+ -- a non-limerick)
Second VARS will get us flowing.
0 gets us where we're going.
n, a comma, p, another
x -- and then it's almost over.
Just make sure that you don't miss
hitting end-parenthesis.
And if you want to see this through
ENTER is the thing to do.

(to find P(X(n,p) ≤ x))
This is something you should love
since it's almost as above.
Only once diverge the path:
Instead of 0, ALPHA MATH.

(Normal random variables)
The standard normal's a wonder.
At the ends it's tossed asunder.
Its mu is none.
Its sigma is one
which is also the area under.

The curve is an exponential
but knowing that isn't essential.
The 83+
is enough for us
to get a good job at Prudential.

(Non-standard normals)
These curves are all wondrous, too.
But they've got different sigma and mu.
Mu is the head
and sigma's the spread
but the area's one, never two.

(Continuous random variables)
It isn't the curves so grandee.
It's the areas under, you see
that do the fine job
of giving the prob
that X lies between a and b.

(normal less-than probs)
As long as big-Z is our venue
go right to your TI menu.

Yes, 2nd VARS 2
will start it for you
and negative 10 will continue.

(normal greater-than probs)
Again, 83 goes with Z.
And again, 10 will join the soiree.
But this time 10's sign is
a plus, not a minus.
Then all will proceed merrily.

(Fuzzy Central Limit Theorem)
Honest, this isn't a scam
that the top of the histogram
(and I do not err)
for "most" X, yes sir
is a normal curve, yes ma'm.

(TI83+ Keystrokes for finding Cumulative Prob's) (Warning: Non-limerick)
If this, instead of Earth, were Mars
we might hit VIRTH instead of VARS.
If Earth were square instead of round
we'd arrow up instead of down.
And if we meant "Return to Sender"
what we hit would not be ENTER.
If our goal were finding z
the next four strokes just wouldn't be.
And if we didn't know which z
we wouldn't hit it, natur'ly.
And if we want to poof this venture
we'll forget to punch in ENTER.
But since none of these is true
we'll do just fine with what we do.

(TI83+ Keystrokes for finding z when we know the cumulative prob up to z)
This might be something we all love
since most of it is like above.
Just, if we think it's much too nice
we'll arrow once instead of twice.
And if we're masochists, indeed
we'll do those four strokes we don't need.
And if we didn't know which P
we wouldn't hit it, natur'ly.
And if we were intent-preventers
we'd forget to punch in ENTER.
All in all, it works out great

with cause, effect, and help from fate.

(finding cumulative prob's of non-standard normals)
(If you're wize / you'll standardize.)
Step One. Just take x minus mu
divide by the sigma. Step two:
Dig into your pocket
(or where'er you stock it)
and with your TI rendevous.

(Central Limit Theorem)
If, ladies and fine gentlemen
we take all the samples, size n
then the set of their means
fulfills all our dreams
for n at least three times ten.

(Confidence Intervals for the Population Mean)
We can never exactly find mu.
That goes for approximate, too.
But the more the dissent
the more confident
we can be; that's the best we can do.

(Calculating Confidence Intervals with the TI83+)
We start with STAT arrow-right twice
and then chose the Z int device
next, put in our data
some this-a, some that-a
and CALCULATE makes it precise.

(Hypothesis Testing -- H-sub-A / leads the way.)
For this very significant lesson
H-A is the thing to obsess on.
H-nought is there
more for the flare.
It makes things a little more pleasant.

(Various "tails")
"A one-l lama is a priest.
"A two-l llama is a beast.
"And I would bet a silk pajama
"there isn't any three-l lllama."

A one-tailed test is a bombshell.
A two-tailed test is something else.
And I would bet a treasure chest
there isn't any three-tailed test.

A shady lady named Nadia
was studying Stat at Arcadia.
She drew legions and legions
of shaded regions
and that made her just a tad shadier.

(The Test Statistic)
Though its various symbols encumber
it really is only a number.
On top goes the gap
on bottom more crap
making us dumber and dumber.

(Rare Event Rule -- "Blame the claim.")
If that test statistic is rare
and H-nought implies it ... well, there
is some explanation
spelling H-nought's cessation
and H-sub-A's premiere.

(Using the TI83+ for Hypothesis Testing)
From our trusty TI83
we squeeze out the value of P.
If alpha is steeper
H-A is a keeper
unless further tests disagree.

Yes, trust our trusty one.
It says do, consider it done.
And if P is foremost
H-A is toast.
(We can eat it with cinnamon.)

We start with STAT arrow-right twice
and then choose the proper device
next, put in our data
some this-a, some that-a
and CALCULATE gets it precise.

Said a Prob/Stat whiz named Ms. Bertie
"The CLT is less sturdy

“so instead of the z
“we use little t
“whenever our n’s less than thirty.”

Guiness had said that he cudent
so Gossett decided he wudent
and so little t
is more famous than he
and bears the proud name of Student.

“Difference between” implies two
and that will affect what we do.
There should be two x
(on the screen and the text)
I think that’s a pretty good clue.

(A little regression session)
The data get listed in pairs
(and we hope there won’t be any spares).
So the first thing to do:
L1 and L2.
(We don’t mind if the data is scarce.)

The screen will show r-square and r.
Both tell us how near or how far
to or from
a straight-line outcome
those points, when they’re plotted, are.

(ANOVA)
H-zero is quite long and lean
a change from the same old routine
with one or two lines
of equal signs
and the mu-sub-i inbetween.

H-A is much shorter and sweeter.
No, it isn’t an equal-sign breeder.
It needs only one
to counter H-none
)and to help with the shyme and the meter).

A fair maid from northeast Andover
said “What’s the big deal with ANOVA?
“It’s mostly a matter

“of listing the data
‘and soon after that it’s all over.”

Said her kind cousin Mary Magee
“Ah, but what of the poor 83?
“It contends right and left
“with test stat. big-F.”
Said the maid, “I’m just glad that’s not me.”

Proportions are good to make friends with.
No sigma or s to contend with.
Just n and p-hat.
It makes for good Stat.
So that’s what I’ve chosen to end with.

(Farewell)
If the pain persists on morphine
try an anti-histamine
or caffeine, or saline
or dentine, or codeine
or an anti-statistics vaccine.

TOPOLOGY

We all know the concept of set
but not of topology yet.
The latter's a class
of the first, but alas
there's more to the story than thet.

E.g., it is quite necessary
that they all be in X, solitary
and that intersects, fin-nite
also be in it
and union, arbitrary.

(Accumulation points)
These are points which a set seems to touch
but not, we observe, very much.
They can get just as close
as your eye or your nose
but never as close as your guts.

(Definition of continuity)
No epsilon-delta in this
since numbers don't even exist.
No fractions, decimals,
infinitessimals.
Instead, we must find a new twist.

Points jump from the old to the new
as the sets they comprise also do.
And so, in this scrimmage
the f-inverse image
of each V must be some U.

Tau from tau.
That's the law.

(Base for a topology)
Hee-haw! Hee-haw! Hee-haw!
A base is a sub-class of tau
whose various u-
nions hit through and through
though it itself doesn't at all.

(Sub-base)
Rub-a-dub, rub-a-dub, dub.
A sub-base is not always sub.
But it is a base
in certain ways
so we welcome it into the club.

If X is discrete (tau not slight)
we claim continuity bright
for any old f
from X (on the left)
no matter what Y's on the right.

And here's something equally deft:
If Y is discrete (not bereft)
then open, we cite,
is Y on the right
no matter what X on the left.

(Homeomorphisms)
Oh, bring out the violins
for here's where the drama begins:
Whither goeth the first
(for best or for worst)
doth goeth the second -- they're twins.

(Metric spaces)
These spaces we truly adore
'cause they provide numbers galore
for us to grab onto
whenever we want to
(just like we used to before).

(Hilbert space)
Square-summable sequences come
with a metric involving square-sum.
And it's isomet-
to a proper subset
which might seem surprising to some.

(first countable)
This means something's countable, yes.
But what? Well, each point must possess
a base local
that is countable

(perhaps more than one, but not less).

(second countable)
Again something's countable, true.
But this time just ONE thing will do
for this kind of space:
one countable base.
I promise, we will not need two.

(separable spaces)
They're countable, in some weird sense
'cause some countable subset is dense
thus reigning them in
agin and agin
making things rather intense.

A fair maid named Mary Michelle
was smitten with Heine-Borel
and her fair cousin Dinah
liked Borel-Heine.
They got on exceedingly well.

A net is a sort of jet-set.
There's no fish that it cannot get.
With an epsilon-pole
it reaches its goal
and manages not to get wet.

(totally bounded)
Hippety, hippety, hoppety!
A set which possesses this property
is finite, of sorts
(by all sound reports)
though by thinking a trifle sloppity.

(The Separation Properties --- Seeya later, separator...)

Heigh-ho! Heigh-ho! Heigh-ho!
Three cheers for our trusty T-oh.
It separates points
all over the joint --
really just one of them, though.

T-one is a nifty go-getter.
It's a tasty trusty trend-setter.
Yep, it's very hep

the way it doth sep-
and it separates them better.

Yahoo! Yahoo! Yahoo!
It's time for our trusty T-two.
It deftly gets
TWO open sets.
Yes, that's what we trust it to do.

(regular spaces)
Here comes a whole different name
playing a whole different game.
It dares to make bets
on points and closed sets
putting the others to shame.

(normal)
This guy does a little duet.
It can take on any two sets
that are diagnosed
as being closed.
It's the best separator yet.

SEPARATION STEW
We learned T-oh
to and fro.
And then T-one
was lots of fun.
We did T-two
fro and to.
Went through T-three
with glitz and glee.
And T-three-and-a-half
gave us a laugh
until T-four
walked in the door.

I'm sure T-five
would really jive
and with T-six
we'd get our kicks.
Lucky T-seven
would be pure heaven.
Likewise T-eight
would be just great.

T-alept-null
would be wonderful.
But we stop at four.
There is no more.

(In any normal space, any two closed sets can be “separated by a continuous function”.)
How very superb and eclectic
to deal with this layout so hectic.
With mere open sets
cool Urysohn gets
all these numbers with nary a metric.

CATEGORY THEORY

(Introduction: “Functions” in the Raw))
O hear ye! O mathematician!
Let us make the momentous decision
to give up x and y.
We can if we try
and still get to keep composition.

(“Arrows only”)
Now, objects are nice and compact.
They stay put and concise and intact.
But the stars of our show
are those things that go
from one to the other and back.

(Discrete category)
What’s wrong with this party?, pray tell us.
No one talking to anyone else.
They stand in their corners
like old Sunday mourners
conversing with only themselves.

(poset categories, non-discrete)
This party’s a bit more grandiose
sporting conversation verbose.
Just one way, true
and one line, too
but at least that one line isn’t closed.

(Larger categories)
Uh-oh, I am getting a hunch
that somebody’s spiking the punch.
That the gang is all here
is most crystal clear
and it’s up to the raunchiest stunts.

There once was a chap from Milano
who couldn’t tell epi from mono.
I told him “get wize
“and just memorize”
but I’m willing to bet he’s not gonna.

(Duality)
Hippity hippity hop.
Ev'ry C has its own Op.
But off C-Op-Op
both Op's we can drop.
Yop, after one Op we can stop.

(Universal mapping properties)
O, what are little UMP's made of?
And what are big UMP's made of?
Arrows galore
and much much more
and that's just what we were afraid of.

(Product = projection-collection)
This hunter shoots many a spear
but just one into each object here.
And any who dare
to likewise fare
must take a quick step to the rear.

(Co-product = injection-collection)
It's the object of this grand old hunt.
(Don't worry; the arrows are blunt.)
And if anything tries
to fare likewise
it must take a quick step to the front.

(Are you an equalizer-sympathizer?)
Its claim to fame is plain.
Its aim is to tame the twain.
Whatever the arrows
how wide or how narrow
so long as both ends are the same.

(Equalizers and co-equalizers)
We can't get these kids to agree.
They fight like Mohammed Ali.
But between Dad and Mom
on both sides, so calm
methinks we can tame the twee.

(Pullbacks and pushouts)
If one end is not the same
we can still pay the game, I proclaim
for a similar deal
reinvents the wheel
and we would be fools to complain.

(Limit)
It's next in our grande repertoire
shooting arrows so near and so far.
And further, t'boot
they have to commute
with the arrows that already are.

(Products and equalizers get us all limits.)
We don't need pullbacks or initials
or ceremonies judicial.
All we will need
to finish the deed
and all we will ask
to accomplish the task
and all we request
to pass the test
and all we require
to light our fire --
yes, all we need seek
are prod's and eq's.
So on we will plod
with eq's and prod's --
to make the whole thing official

(Functors)
Here they come, marching in droves.
Both the contrav's and the cov's.
Some imbed
and some forget
and some we don't yet even know of.

(Adjoints: Hom (FC, D) ≅ Hom (C, UD))
It's Old Hom Weekend, it seems.
And in fact there are TWO Hom teams.
But, except in name
those teams are the same
if we take matters to extremes.

(F and U)
Things would turn out mighty rotten
and F would feel pretty downtrodden
and most disappointed
because un-adjointed
if forgetful meant also forgotten.

(Seeya later, Yoneda)
Any C can be co-completed.
Small-y is all that is needed.
And if there's a goof
somewhere in our proof
don't worry, we'll simply delete it.

(Endo-functors and P-algebras)
Endo's are sprightly as minnow
and they endo where they doth begin-o.
Their activities buzz
with P-algebras.
Get ready for that to contin-o.

(Finale)
See, there's life after x's and y's.
Did you savor the fun and surprise?
Did you like how it works?
And relish the perks
like no compound fractions or pi's?

METRIC SPACES (in progress)

It is time to consider existence
of a gen'ralization of distance.
These functions so binary
reveal their vast finery
to any kind soul who listens.

From here to here is squat.
From here to there is not
though the same,my dear
as from there to here.
Stay tuned for a thicker plot.

Getting between here and there
then on to everywhere
has got to be more
if we know the score
than as the crow flies through the air.

Good news! We're already aware
of the root of the sum of the squares.
True, if we'd our druthers
there wouldn't be others
but that wouldn't be very fair.

Three cheers for the matrix inverse.
It makes tiny numbers disperse
and the big ones converge
in a general purge.
With the middle ones not much occurs.

(Oodles of others)
Metrics take many a form.
Some of them come from a norm.
Some are discrete.
Some are complete.
Some are quite trivial.
(We think them convivial.)
Some are extensions.
(It can't hurt to mention.)
Some are restrictions
(as per our predictions).
Products, p-adics
(for all us fanatics).
Functions injective

(They've been most effective.)
-- and l-p will make it a quorum.

We will now run the gamut -- and gamut-er
by generalizing diameter.
We might not be pro's
but as far as this goes
we'll be pretty impressive amateurs.

(distance from a point to a set)
Instead of the greatest -- supremum.
Instead of the smallest -- infimum.
"Cause the points that we need
important indeed
are such that we might have to dream 'em.

We have distance from sets to sets
but that is as far as it gets
'cause the distance selected
might not be a metric --
nope, no Hausdorf metric yet.

A point's in the boundary of S
if its distance is zero (or less)
from S and S-comp
so without further pomp
we'll just say it's an easy access.

Keep losing those poor middle thirds --
Is anything left afterwards?
Yes, the cool set of Cantor
is quite an enchanter
although it might look a bit blurred.

(Hausdorff metric)
For e-ve-ry metric one gets
a metric on closed bounded sets.
And the twain doth meet
when it comes to complete
and neither has any regrets.

When a metric space waxes compact
we can harness another cool fact:
Compact equals closed
and that,my friend, goes
on both a P.C. and a Mac.

A space has a nearest and dearest
when it is complete,and the merest
of bounded subsets
even more bounded gets.
Or so would assert any theorist.

If f is in C-S-R
where S must be closed (not ajar)
then the itsy bitsy
theorem of Teitze
extends f to some f-bar.

Some folks can get hooked on phonics
and some have a thing for bionics
and some have a yen
for vodka or gin
and some have fixations on Banach's.

I don't mean to be too outspoken
and I hope you don't mind my sly jokin'
but how can omega
be fixed, I dare beg ya
without having ever been broken?

Think of open dense sets so elite
and I'll tell you a secret so sweet:
each countable c'llection
has dense intersection
assuming that X is complete.

PROSE BACKGROUND: Once, while teaching a class, I had occasion to say "shift". -- "What?!" exclaimed a few students. -- "Don't get all excited; I said 'shiFt.' " -- "Oh." They seemed a tad disappointed. ("Hey," I quipped, "are you-guys trying to get me fired?") So -----

This time I will say as desired
and pronounce this guy's name as required.
It is not LipshiFtz.
It is Lipshitz.
Yup, this time I'm gonna get fired.
(And we can just say I retired.)

FERMAT'S LAST THEOREM PROVEN

Fermat said the proof was too large
to fit in the right or left marg-.
True, back of the paper
or proof made to taper
might help, but he said, "I'm in charge".

Now, Wiles didn't mind paper waste.
In fact, it was true to his taste
to use up whole reams
to realize his dreams
and he crossed out instead of erased.

Fermat was all snickers and smiles
as he smugly stayed clear of the aisles.
He thought "they'll be glum
"but that proof will succumb
"though it's going to take quite a-Wiles".

A COMPUTER NON-LIMERICK

A one-r erol is a strain.
A two-r errol is a pain.
But we would be in greater peril
if there were a three-r errrol.

HISTORY OF MATHEMATICS LIMERICKS

Three cheers for the Old Babylonies.
We love those illustrious cronies.
Instead of our decimals
they used sexigesimals
to count up their stickies and stonies.

(ancient Chinese math)
Fractions, zero, and scanter
plus geometrical banter --
You name it, they'd do it.
They beat Euclid to it
and Euler and Gauss but not Cantor.

("adding and doubling", ancient Egypt)
There was a fair maid from Versailles.
They say she could not multiply.
Well, only by two
so that's what she'd do
and that was enough to get by.

Myra has been most desirous
of mast'ring the Rhind Papyrus.
But to read the inscriptions
she must learn Egyptian
and that's not a favorite of Myra's.

(reality check)
As these poems you so dutifully edit
keep one thing in mind, to my credit:
I realize "inscriptions"
is plural, while "Egyptian"
is singular -- well, let's not sweat it.

Egyptian math is a ball
both the large of it and the small.
The main attraction:
Egyptian fractions
which really encompass them all.

(600 AD)
All hail to those wonderful Hindus.
They knew all their sums and gozinto's
and square and cube roots
and other pursuits
and they did it all without Windows.

(reality check)

As these verses you carefully edit

keep one thing is mind, to my credit:

I realize that “Windows

does not rhyme with “Hindus”

but I have no plans to regret it.

(ancient Greek unsolved problems)

You think you’ve got big troubles?

Well, the Greeks had the cube to double.

And the circle to square

the angle to tear

and Euclid’s Fifth in the rubble.

(Euclid’s Fifth)

Two lines cross a third, in great freedom.

What side do the first two lines meet on?

Why, the one with interior

angles inferior.

If you want to learn further, then read on.

(Reality check)
As these lines you so suavely inspect
please treat them with ample respect.
Yes, certainly read on
does not rhyme with freedom.
So? Surely the math is correct.

Other forms come by the wayfare
like the parallel one from Ol' Playfair.
Through a given point
to a line so disjoint
one parallel, found by lassez faire.

Triangles give us another
form of the Fifth to discover.
It's pretty abrupt.
The angles add up
to one-eighty, not under, not over.

(reality check)
As these lines you so caref'ly go through
keep one thing in mind as you do:
I realize "discover"
does not rhyme with "over"
but that is the best I could do.

I would swear on a stack of Gideons
that Euclid was very euclidean.
His lines were straight
his great-circles great
whether or not meridians.

(reality check)
As these poems you so caref'ly assess
for accuracy and finesse
you'll notice that "ideans"
is not quite "idean".
Well, true, but I did my best.

A most heartfelt toast to the Greeks
their cliques and techniques and mystiques.
They pushed their careers
for one-thousand years --
that's fifty-two-thousand weeks.

An equation solver named Tantis
was as picky as Diophantus.
'Twas his worthy goal:
x had to be whole.
Anything else was outlandish.

Omar Khayyam lived ‘way back when.
Both poems and math were his ken.
Some say he was two.
I can’t say I do.
But I wasn’t there, nor then.

All hail to our friend Fibonacci.
He tinkered like ol’ Liberaci.
He started off slow
two one’s in a row
but ended up vivo vivace.

There once lived a young lad named Bart
quite versed in the old cossic art.
He wrote p for plus
and m for minus
and that wasn’t bad for a start.

No one had yet solved the cubic
which, along with the quartic, seemed too big.
But then came Cardano
who said “Man! I’m gonno.
“Whaddaya think I am -- stupid?”

(reality check)

As these poems you so dutifully edit

keep one thing in mind, to my credit:

I realize that “stupid”

does not rhyme with “cubic”.

Whaddaya think -- I’m thick-headed?

MATH WARS

They fought, though not with swords

for credit and awards.

Their rip-roarin’ wrath

was how they did math

(and passed the College Boards).

(Cardano vs. Tartaglia)

They fought, though not with knives

for the mathgem of their lives --

the mighty rubric

for solving the cubic --

and only the math survives.

(Newton vs. Leibniz)
They fought, though not with rifles
over itty bitty trifles
like dx and dy
and Lord knows why.
I guess they enjoyed being spiteful.

(Reality check)
As these lines you so thoroughly scan
keep one thing in mind, if you can:
I realize that spiteful
does not rhyme with trifle
but that goes along with my plan.

(Kroenecker vs. Cantor -- Kroenecker is famous for saying, “God made the integers.
The rest is the work of man.”)
They fought, though not with blades
o’er which were the numbers God made.
K. thought the whole ones
to be the sole ones
but C. had a whole long brigade.

A most loyal toiler was Euler.
He left nothing on the back boiler.
He was mostly blind
but he didn't mind.
No, that wasn't an Euler-spoiler.

(l7th century unsolved problems)
You think you've got great big woes?
Well, they're nothing to Cardano's.
His formula veered
towards numbers weird
  and Leibnitz and Newton
  kept on disputin'.
  and no one knew
  how to write "equal to".
  and the proof that Fermat
  so deftly forgot
and on and on it goes.

(polynomials)
One could deal with degrees one to four
but not with degrees five or more
and neither could Abel
(not even with Maple).

Still, he was the first to know that for sure.

(reality check)

As these poems you so dutifully edit

keep one thing in mind, to my credit:
I know that last rhyme

took ‘way too much time.

It got just as far as I let it.

Primes come in all different hues.

It’s hard to remember who’s whose.

Fermat’s, Mersenne’s

or Sophie Germaine’s.

But they all had to do with 2’s.

(Helter skelter, Kronecker delta)

It’s a function of i and j

as simple as night and day.

Sometimes it’s one

most times it’s none

and there isn’t any half-way.

Ring-a-round-a-rosy.

Cauchy, Cauchy, Cauchy.

Cauchy this

and Cauchy that.

Google if you’re nosy.

(Reality check)

As these words you so carefully weigh

and notice some rhyme disarray

specifically, nosy

does not rhyme with Cauchy

... well, google him anyway..

Oh, bring on the nineteenth century!

Math became less elementary.

Sophistication

professionalization

and rigor made things more adventure-y.

(Reality check)

As you go through these limericks cool-ly

you’re so conscientious, truly

so you’ll see that last word

is kind of absurd.

I’m sorry to be so unruly.

(Also, here’s my chance to mention the brothers Bernoulli.)

As he painted square-inch by square -inch
someone asked Leon di Vinc
“You draw so projective
“and so non-defective.
“That hard?” He replied, “It’s a cinch.”

For Peano success meant successor.
The number succeeded was lesser.
And the one that succeeded
was urgently needed
to make him a full professor.

(l9th century unsolved problems)
You think you’re a goner-to-be?
Think of ol’ Riemann’s zeta of z.
And the primal pain
of Sophie Germaine
  and the par’dox colossal
  of Bertrand Russell
  and the mental block
  of poor Goldbach
and then Hilbert’s mean twenty-three.

(reality check)

As these pages you so wisely nitpick

keep one thing in mind as you flick:

I realize “colossal”

does not rhyme with “Russell”

but it’s better than “large” or “big”.

(early 20th century England)

When Hardy stumbled upon

the amazing Ramanjuan

his joy knew no bounds

at whom he had found

and together they worked on Riemann.

(20th century trends in the mathematical community)

More journals, more meetings -- just more

of what there had been before.

The fields got connected

and politics corrected

and women were let in the door.

(Two party poopers -- c/o Yolanda)

Russell made sets seems fictitious

and Godel made axioms suspicious

and thus did we

learn uncertainty.

(I think that makes math more delicious.)

(Two party animals...)

May I offer the following hunch?

(We could argue it over lunch.)

To say that these two

were poopers -- not true.

They merely spiked the punch.

(non-limerick about Paul Halmos's coining of the word "iff", meaning "if and only if")

A one-f if goes just one way.

A two-f iff is interplay.

And I would bet a hieroglyph

there isn't any three-f ifff.

In class we had quite a grand gala

with that cool ancient game called mancala.

We sure lived it up

as we took from each cup

and dropped in the ones to follow.

(Reality check)

As these poems you so fondly devour

utilizing your editor-power

observing that follow
does not rhyme with gala,
I hope you don’t charge by the hour.

We also were far from grim
as we spent more time playing nim
than our breaks would allow
and we’d do the same now
with time to spare for a swim.

And when our breaks were all broke
we took a few secs for Sudoku
with computer or pen,
and we’d do it again
with time to spare for a coke.

(still-unsolved problems)
You think you’re got trials and tribs?
Well, Goldbach has first dibs.
Are there enough evens
to still his a-grievin’?
    and enough twin-primes
    to last for all time?
    and irrational powers --
    will they ever be ours?

Will conjectures turn out to be fibs?

In this modern uncertainty trend
one thing is certain, my friend:
With the subtle hurdle
c/o of Kurt Goedel
the history of math will not end.