MATH-TEACHING LIMERICKS

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MATH 101

Functions are just like numbers.
They get lugged around like lumber.
They get add- and subtracted
and more interacted
making us dumb and dumber.

(Cost Analysis)
The cost of producing is mixed.
Part of that cost is fixed.
And the rest, never-ending
on level depending
(or maybe on politics).

(Rational Functions)
Said your favorite Zada or Tante
“for an asymptote horizonta
“you need to use both
“of the leading coeff’s
“whether or not you wanta.”

(Exponential Functions)
The bottom is perfectly noble
but x is now upwardly mobile
so that constant, poor dear
must downsize it’s career
and it’s feeling a bit claustrophob-al.

These guys are as busy as beavers.
They’ve got Monday morning fever.
They just grow, grow, and grow.
They’re NEVER zero.
And they don’t remain one for long, either.

Do logs give you logarrhea?
Is a logjam drawing near?
Just use your credentials
and take exponentials
to make those ol’ logs disappear.

(Matrix multiplication)
Just pair off each row with each column.
Take the sums of those products so solemn.
Yes, beat the odds
with those scalar prods
or whatever the devil you call ‘em.

(Compound Interest)
“Nothing succeeds like success,”
as these formulas show with finesse.
Thus again and again
A gets bigger with n
though not quite as big as you’d guess.

(Annuities)
And now here’s another fine source
(to tickle your brain in this course)
of mo’ and mo’ dough.
It’s dough a go-go.
Oh, don’t you just wish it was yours.

CALCULUS LIMERICKS

Diff-ing x-to-the-n is fun.
Change the n to n minus one.
Then go back to n
and use it again
in front of it all, and you're done.

We can diff any power of x
on this whole blasted campus of Drex.
n can be miniscule
as an H-molecule
or as large as Tyranno Rex.

It is not my intention to vex.
It is not my intention to hex.
My only intention
is merely to mention:
The diff of x-square is 2x.

It is not my intention to glare.
It is not my intention to scare.
My only intention
is merely to mention:
The diff of x-cube's 3x-square.

There's a general rule for all this
(in case someone pulls a pop quiz):
n steps down two ways
for the rest of its days
and x remains right where it is.

"Than receive it is better to give."
Also, "live, live, and let live."
These are words to clutch
but not as much
as "velocity equals deriv".

(General Power Rule)
Are you bored with powers of x?
What would you like to do next?
Some powers of g
will do nicely
but watch out for the special effects.

On, Dasher! On, Dancer! On, Prancer!
Don't forget the power in the answer.
It gives it might
and makes it right
also a little fancier.

Three cheers for parentheses!
They make forests out of trees.
But be careful about
what's in and what's out
or they'll look like they have some disease.

(Product Rule)
Don't forget -- f appears twice.
g will behave likewise.
If you make like a dunce
and write them just once
... well, you'll get what you get for half-price.

Said a wiseguy named Georgy O'Porgy,
"Let's have a Quotient Rule orgy.
"On top, to be nice
"let g appear twice
"and then on the bottom one more g."

(Chain Rule)
Rinky, dinky, dinky.
g provides the link-y.
But the x and the f
are not to be left
unless we run out of ink-y.

Ln x, you can't just be.
You have to get diff'd, yessirree.
So give ln the slip
then take the recip
to get one-over-x, easily.

(General Ln Rule)
On bottom goes the copy.
And what goes on the top-py?
Why, the diff, g-prime.
It makes it rhyme.
And please don't write it sloppy.

(Integrating Powers of x)
There's a general rule for all this
(and it's something you don't want to miss):
n moves up and down
all over the town
and x stays right where it is.

(Integrating Exponentials)
And now here's a grave admonition
delivered with proper precision:
It's about that k.
It steps down just ONE way
and the x doesn't go where it isn't.

Minus-one is a cool special case
delicious and dainty as lace.
So don't play the hero.
Don't divide by zero.
If you do, be sure to erase.

(Curve-sketching)
We can plot and plot 'til we plotz.
But we've got to plot the right spots.
Or those lows and those highs
could elude us like flies.
Likewise the flips and the flops.

There was a young man named Kareem
who explained, "For a local extreme
"the tangent at a
"to rest must lay
"and we wish it the pleasant-est dream."

Said his kissin' cousin Trix
"But not vice versa -- nix.
"That tangent can flatten
"as low as Manhattan
"with no min, no max, just a mix."

A fair maid from North Minnesota
was drawing a steep asymptot-a.
When it got 'way too high
she murmured bye-bye
and mourned not a single iota.

(Integration by Parts)
"We need f and g-prime," said Mitch
"and it matters which is which."
"But not to worry,"
said his cousin Murray.
"It doesn't work out, we'll switch."

Root-a-toot too-a-falutin'.
It's time for some substitutin'.
Take stuff on display
and collapse it away
right along with Leibniz and Newton.

The differential
is essential.

(Area between Curves)
If the graph of x-to-the-sixth
with x-to-the-eighth is, not mixed
only placed, for our practice
on the same set of axes
then subtract to find what's in betwixt.

Don't forget, those curves might cross
in which case you should take time to pause
so you don't subtract
when add's where it's at.
'Twould be a lamentable loss.

To split or not to split?
'Til the question that stymied Hamlet.
But when you're in doubt
don't leave the split out
though it's harder, I admit.

We could sit all prim and proper
privileged as gold and copper
but 'twould perk our careers
if we'd get off our rears
and become intersection-hoppers.

(Solids of Revolution)
A strapping young woman named Evvie
was handed a solid of rev-y
and asked for the volume.
She answered, quite solemn
"it's not very big but it's heavy."

(Average Value of a Function)
Riki tiki tavi.
Here's some calculus savvy.
The inteGRAL
o'er the interVAL
will give us our function's av-y.

(Integration Techniques: Sine-Saving)
Said a technique freak named Zeek
"If you think sine-saving is chic
"and you want to save
"yourself into the grave
"try cos and tan and sec."

Here's a little ditty
helpful if not cute:
What we save is the deriv
of what we substitute.

(Deriv's of Trig Functions)
A darling named Clementine
said, "First sine, then cos, then sine.
"And the minus and plus
"make things even wuss.
"Can't it make up its mind?"

A Non-Limerick:
Can we integrate tan?
Yes, we can.
Can we integrate secant?
No, we can't.

(Trig-triggers -- i.e., trigonometric substitutions)
A trig-trigging trickster from Beacon
is stuck on an odd-powered secant.
An integral table
would render him able
but his conscious is prodding "no peekin' ".

Trig, trigger, triggerest.
Try not to be too vigorous.
If you trig too much trig
it'll get too big
and you'll fall down just like Icarus.

(l'Hospital's Rule) (Good Golly, Miss Molly! Here comes l'Hospital-y.)
There was a fair maid from Nepal
who was expert in ol' l'Hospital.
She diff'd 'til she dropped
on bottom and top
then murmured "Nice knowing y'all."

(Hyperbolic Functions) (Are you a hyperbolic-aholic?)
Take the trig I.D.'s, all kinds
add h to those cos's and sin's
and this derring-do
leaves them all still true
except for a few miinus-signs.

(Completing the Square)
When the middle term nerdily lurks
use this to get rid of the jerk.
Sometimes a wild guess
will get rid of the mess.
And algebra always works.

(Approximate Integration)
The Trapezoid Rule can be fun.
All those 2's will get the job done.
But watch out, my friends
for the left and right ends.
At those we will need only 1.

Cheer up. (Don't look like such wrecks.)
I know this seems pretty complex.
And there's many a y
where the midponts lie
but there's only one delta-x.

Just look at those sixes and fours!
They're the prettiest things in this course.
And this cool rule of Simpson
will stay sweet and winsome
once you've become sophomores.

(Improper Integrals)
Said a ship-shape chap from the Congo
"Since our region is infinite long-o
"it's likely that you'd
"be inclined to conclude
"that it's infinite big, but you're wrong-o."

Said a dashing young bloke named Apollo
"Said our region is infinite tall-o
"It's likely you'll dig
"it be infinite big
"but sorry, that doesn't quite follow."

(Separable Differential Equations)
A ditzo from hither or thither
refused to get all in a tither.
"'Tis true," said he, "I
"can't tell x from y
"but can't tell left from right-y, either."

The vertical line test's a pity.
It stops curves from being pretty.
But we can make loops
without saying "Ooops?"
with parametric graph-iti.

(Polar Coordinates)
With these we can also be arty.
We can have a plotting party.
While away the hours
making petals and flowers
along with Mercenne and Descartes.

Would you like to come up to the board
and draw r and theta coord's?
We'll get hulas and hoops
and loop de loops
and other delicious rewards.

(Infinite Series -- Comparison Test)
Smaller than small is small.
Taller than tall is tall.
And that's how it goes
and that's how one knows
whether anything happens at all.

(Integral Test)
If we know what happens with n's
all x's will follow, my friends.
It's all essentially
the same, consequentially
and on each the other depends.

(Alternating Series)
However they rageth and roareth
and wobbleth back and forth,
you'll eventually find
that they make up their mind
someplace between south and north.

(MacClaurin Series)
The first in our fine repertoire
is one over one minus r.
We mustn't forget it
for where we are headed
or else we won't head very far.

An industrious lad from the near East
was summing a long Taylor series.
At the twentieth head
he just shrugged and said,
"That's as far as I go, my dearies."

(Partial Deriv's)
We x 'em and we y 'em.
But not both at the same time.
f_x is one
f_y's also fun
though we never get to f-prime.

(End-of-Term Farewell)
Yes, I know, it's 10:49.
But this proof needs one more line.
I promise I'll write
with all of my might.
(Keep mum if I'm off by a sign.)

COMPLEX ANALYSIS

A reality complex had Fred.
He felt kind of bad in the head.
So he took Complex Var
with Cohen Mar
got a complex complex instead.

There was a young lady named Suzie
who couldn't add 2z and 2z.
She said, "Can't it be
"without that ol' z?"
She's being a little too choosy.

(nth Roots of Unity)
There are n ot them sprawled on a wheel.
Among them at most two are real.
The others must go
half above, half below.
But they get paired off in the deal.

e-to-the-z is exotic.
And it changes a sum to a product.
It has no root
but it's kind of cute
and it's vertically periodic.

A young man named Kenny macKenzie
had trouble computing ln z.
He breezed through the r-part
with the slickness of pop art
but the theta-part gave him a frenzy.

Here's the key to z-to-the-c:
It's just e-to-the-c-ln-z.
But watch for ln-y.
Its values are many
though sometimes e makes them agree.

(Said Sin and Cos)
"In trig and in calc, all through
one was the limit we knew.
But now we're set free.
Now we can be
anything we desire such as two."

How smart the conditions of C-R !
They're almost as brilliant as we are.
But they're known for their sly-ness.
Watch out for that minus
or we'll spend half the night in the E.R.

Yes, here they come, hot off the griddle
spiffy and spicy and little.
One sports a plus
the other mi-nus
and none of them sports the middle.

The research team Cauchy and Goursat
was busting its brain and its torso.
They went huffin' and puffin'
but came up with nothin'.
Now they're living on noodles and orzo.

The pretty professor from Jersey
just stood there and pleaded for mercy.
She said "in complex
"v-y equals u-x --
"or maybe it's vice vers-y."

i after e after u
after pi after n after 2
after p'renthesis ln
and then no more spellin'
unless we can find something new.
(Thanks to Bob Blackard for getting me started on this one.)

Our pretty professor's a rarity.
Her lectures the height of clarity.
Except that when
she does minus-one to the n
n turns out to be the wrong parity.

The pretty professor, our rarity
was engulfed in hyper-hilarity
'cause again she had spun
a spare minus-one.
She said, "Well, we can give it to charity."

(About $\oint f(x)/(z-z_o)^n$)
When C's end is at its beginnin'
ask "Is z-nought out'n or in'n ?"
Draw the point, draw the curve.
Then stand back and observe.
If in doubt get a second opin'on.

Computing it's easy as pie
(especially if you try).
There's a derivation
and an exclamation
and don't forget two pi i.

This classroom is water- and sun-proof
and poison- and noisin'- and gun-proof.
It's as pure as the Gospel
of everything poss'ble
except that it's not minus-one-proof.

Geometric ser's are a gem.
But let's add that little m --
OUTSIDE, if you please
the parentheses.
Did you hope I'd forget about them?

The last of those spunky Mohicans
was at work on a long Cauchy sequence.
It was i - pi - y
from Mon to Fri
and he took it home on the weekends.

Can you count? Then here's one for you.
One less z to the minus-two.
Write 1, 2, 3
then blank, z, z
and the powers and the plus-signs, too.

A fair maid from North Miami
was at work on the triple-whammy.
She tried subtraction
and partial fractions.
Then she called for her pappy and mammy.

A lazy young lad named Laurent-o
used his series to integrate pronto.
Just b-one will do
and as for b-two
you need only find that it you want to.

O what will the fair miad named Tessa do?
She's been after a simple-pole residue.
She says, "NOW I see.
"It's q-prime UNDER p.
"Then I plug in z-nought -- or I guess I do."

TEACH-TONE PHONE (Remember, I gave out my phone number? Well, no one ever called, but if you had, here's what my recording said -- See what you missed?)
For Cauchy-Goursat
press O.
For some integral fun

press 1.
For a residue
press 2.
For I / (I - z)
press 3.
For contours galore
press 4.
But to find out the fate of a minus sign
be patient, please, and stay on the line.

LINEAR ALGEBRA

Allow me some words of great wisdom
about any linear system:
It is represented
by a matrix, augmented
which explains this course's existence.

Do row op upon row op.
Keep doing them 'til you drop
or until rectangular
becomes triangular.
(And the triangle's on top.)

The form which is just-plain row ech-
means surely you'll still have to thresh
with back substitution
so keep on reducin'
to get x-sub-i in the flesh.

(Matrix multiplication)
Just pair off each row with each column.
Take the sum of those products so solemn.
Yes, beat the odds
with those scalar prods
or whatever the devil ya call 'em.

(Definition of Determinant)
O, nothing could ever be finer
than replacing A major with minor.
Of course, for most n
we must do it again.
And the signs will get sign-er and sign-er.

(det A x det B = det AB)
I don't mean to cause you to squirm
but the computations confirm
that determ before prod
(in the eyes of God)
gives the prod before the determ.

When we wish that our fingers were toes
and we wish that the columns were rows
and the i's and the j's
are turned the wrong ways,
'tis time to bring on the transpose.

(how to find A-inverse)
On cofactors get yourself versed.
But start with the transpose first.
And do not forget
to divide by det
and you'll wind up forever cursed.

If you know how to cut and to paste
and can figure determ's with great haste
and one more aside:
if you've learned to divide
then by Cramer's cool rule you are graced.

(to decide whether a given set of functions is linearly independent)
Line up all the functions in sight.
Then diff them with all of your might.
Then sing a sweet song
of right and of wrong
as we get that ol' Wronskian right.

A young man named Timothy Tigen
was searching for values eigen.
But the characteristic
and other logistics
were such that he needed to try 'gain.

(how to diagonalize)
E-values will get us big-D.
E-vectors will get us homefree.
Yep, string up the specs
and line up the x
and we'll get where we needed to be.

If its set of e-vectors is sizeable
then A is diagonalizable.
And the converse, too
has been tried and true.
(I hope that is all recognizable.)

(how to exponentiate a matrix)
Proceed as the previous verse
has instructed (for better or worse).
Treat D as you're tempted
and X is exempted
except we'll still need its inverse.

(Definition of vector space)
We plus 'em and muss 'em and less 'em.
We shrink 'em and stretch 'em and press 'em.
And still they will be
inside our big-V.
Such fine loyal souls, the Lord bless 'em.

(subspaces)
We can push and pull and stress
and otherwise make a big mess
and they'll not only be
iniside our big-B
they'll also remain in big-S.

Can we get infinity from fin
(without committing a sin)?
Oh yes we sure can
with the kind help of span.
(After all, it's the past tense of spin.)

(dimension)
Each basis goes by its own name.
Each vector goes by its own aim.
But if you would count
the number-amount
the answers will turn out the same.

Base B can be used to express
any member of V or of S.
But the thigamabob
that will pull off the job
depend on which B we possess.

To get thigamabob for C
from thigamabob for B
will require the rendtion
of matrix, transition
and seldom will they agree.

(linear operators)
They're linear, down to the core.
They take zero to zero, no more.
And sums and doubles
and all other troubles
to what they had been before.

(Multiplication by any matrix is a linear operator.)
Said a fair young maid named Alexis
"Ax will double if x does
and A takes a sum
into something hum-drum
and there won't be any more extras."

(rank)
A matrix has n column-vectors.
Indeed, it's a vector collector.
It's all touchy-feel-y
but how many, really
are there and how many are exter?

(Column rank = row rank)
You can put them in lines or in layers.
And then, if you say the right prayers
it won't matter which.
The quarter-turn switch
won't change what's essentially there.

(Every linear operator = left multiplication by some matrix.)
Just take what L does to each e
and line them all up merrily.
And that's good way
to get your big-A.
(Watch out so you don't get a B.)

Let's all do a little Gram-Schmidt
get orthogonal vectors that fit.
At every stage k
I'll show you the way.
(At k = n we can quit.)

VECTOR ANALYSIS

Here's something we might contemplate:
A vector's an arrow so straight.
For forces, its length
expresses its strength
and direction expresses its fate.

Two vectors, whatever the size
can be added, component-wize
so make so this rhymes
by adding three times
then put in the k, j, and i's.

(Scalar Multiplicatoin)
A spunky young woman named Ralpha
would muliply vectors by alpha.
She thought it quite nice
to multiply thrice
in this city of Philadal-pha.

(Scalar Product)
Here's the most fun you ever had:
Take products three times and then add.
Yes, do mind your mommas
and DON'T put in commas
(or momma will tell your Dad).

(Vector Product)
In this glossy and glorious firmament
nothing's more perfect or permanent
or more fancy-free
for u criss-cross v
than our ever-lovin' determinant.

i, j, and k are cute
and they're specially fixed to compute.
E.g., i cross j
is lower-case k.
but as arrows, they're too short to shoot.

(Some curve parametrization hints)
Straight lines are quite matter-of-fact
if you know how to add and subtract.
True, there's multiplication
in this situation
but nothing on which you need act

Although we are all mere amateurs
iin this business of making parameters
when it's circles in question
I have a suggeston:
Try not to forget the diameters.

(Formula for Curvature)
r-dot dot r-dot-dot.
Yes, sometimes we dot a lot.
But dot-dot-dot?
Or r-quad-dot?
. . . Well, sometimes we do not.

For that curvature for t' compute
that formula's really a beaut.
But when figuring kappa
do mind your Pappa
and don't you forget that square root.

It's easy and fun to do grad.
It makes us feel gleesome and glad.
Don't you feel so alive
when you partial-deriv?
But put commas, now -- don't you dare add.

Here's something no one will object to:
When doing the div please expect to
takes partials, three
of componenets of v
and be careful which with respect to.

Aloha, tres bien, and shalom.
May I mention a vital syndrome?
Towards the end of the div
you should get additive
(or just wait 'til your father gets home).

Now, the hardest is curl, we agree.
But let's have a determinant spree.
First, three vectors little
next, del in the middle
and then on the bottom goes v.

All told (and telling it true)
there's more in the curl to do.
We've got partials six
and note how they mix
and the answer's a vector, too.

(Directional Deriv.)
If we're out in the wide blue yon
rates of change depend, which directi-on.
So divide b by mag
then dot it with grad
and we'll find out how quickly we've gone.

(Unit Normals to Surfaces Given in the Form f(x, y, z) = C)
Said a dashing young man named Aeneas
"I've got a few dandy ideas:
"Just take our f's grad
"divide by the mag
"and it doesn't much matter what C is."

(Reality check)
Whenn these poems you so dutif'ly edit
keep one thing in mind (to my credit):
I realize that mag
does not rhyme with grad
but the point is: YOU won't forget it.

(Surface Parametrization Hints -- Plane)
If you want to be this term's winner
remember, the keyword is linear.
So let us let z be
ua + vb
(for integrals outer and inner).

(Cylinder)
First, think of the xy-plane.
(That's not such a drain on the brain.)
Next, move once around
and then up and down.
If tired out, go hop on the train.

(Sphere)
Said a student from South Singapore
"We use cosines and sines galore
"or the sum of three squares
"would give us nightmares --
"Yes, that's what the trig stuff is for."

(Line Integrals)
And now we've got functions three.
What could the integral be?
The answer, essential:
Component, tangential.
It gets things in terms of small-t.

(Surface Integrals)
Next, functions (three) with a surface
(And Murphy's Law gets more Murfish).
But the answer, informal:
Component, normal

to get u's and v's at our service.

(Triple Integrals)
Now, triple int's can be quite chic
so of them we'll now start to speak.
But one thing clear:
No vectors here
(or you're grounded for a week).

(Divergence Theorem)
Next, the surface int. above
that we've all come to know and love
will be giving 'way
on this fabulous day
to a triple int. of the div. of.

(Stokes Theorem)
And now it is late, not early.
'Tis time for Stokes' Theorem, surely.
Any int. (closed line)
can be redesigned
as an int. involving the curl-y.

(Reality Check)
Again, as these lim'ricks you edit
remember one thing (to my credit):
I know that designed
does not rhyme with line
but it rhymes more than how Stokes said it.

DIFF EQ

A fair maid from Northwest Virginia
was solving a first-order linear.
But that exponential int.
was too large to print
so she just made it shorter and skinnier.

A starry-eyed lad from Wisconsin
was hoping those coeff's would be constant.
But his hopes turned to hexes
at all of those x's.
He shrugged and remarked "Stuff and nonsense".

There was a young woman named Sally.
Legendre was right up her alley.
"Minus-x-square plus one
"is what starts the fun
"and n-square plus n's the finale."

Her cousin named Mary Magee
said "Bessel seems besser to me.
"x-square WITHOUT one
"seems much more fun
"and nu's cooler than n," said she.

Their cousin from Doodle-Skadiddle
said, "Yes, but what's in the middle?"
Answered they, "we suspects
"it's x or 2x.
"We're feeling a big non-committal."

An attractive young ms. named Cassandra
was solving equations Legendre.
a-nought was do-able
a-one construable
but a-(n+2) was beyond 'er.

There was a fine fellow named Ian
who was ready to give up on p_n.
He said, "Too much mess
"with the n and the x".
And sometimes I can't help agreein'.

Let's do J_n versus P_n,
The former's a much smaller bein'
with much less subtraction
and all told, less action
'cept: more plus-signs in between.

We've y_1 but not y_2.
What're we gonna do?
Just look to the board
at Red. or Ord.
how, instead of C, we try u.

We've y_1 and y_2 sublime.
But r 's not zero this time.
So do Var. of Par.
with Cohen Mar
then stand up and drink to l'Hyam.

(Picard's Successive Approximations)
Again and again and again
from n-1 to n.
We get closer and closer
(If it's cos, we get cos-er)
but making no promises when.

(Euler-Cauchy Equations)
They sport both dx and dy
and an x-square and a by.
And the thing to do next:
guess a power of x.
When you plug it in, you'll soon see why.

(Bessel Functions)
In these guys the x appears twice
'cause the powers get strategically sliced.
The x that is mighty
goes to the right-y.
To the left goes the x that is nice.

PDE

A lad thought he'd service society
by studying math and psychiatry.
But his plans went a-ragin'
with the first wave equation
'cause he got Separation Anxiety.

Four, Fourier, Fouriest --
Isn't it just-plain glorious?
How, from sin's and cos's
we get (by osmosis)
so much, and emerge victorious.

(Fourier Series in Exponential Form)
Those functions e-to-the-in-x
can be treated like cos x and sin x.
And if they can't
call me Bell Atlant-
or try MCI or Ninex.

(Fourier Integral)
Are we feeling all morbid and mopey
just 'cause f(x) has no p?
Don't be so timid.
Go to the limit.
When there's life, there's always hope-y.

It's Jan. 14 and our prof
by a factor of TWO is off.
Is she going to cry?
Says she, "No, not I.
"In fact, I'm more likely to laugh."

Do you think this is getting hum-drum?
It's time to consider a drum-drum.
We'll get double series
and double theories
and B's and B*'s under 'em.

Next we make our rod very lengthy
thus increasing its impact and strength-y.
So we've lost the L
but gained integrell.
For thy patience, I humbly thank thee.

There was a spry guy from North Saigon
who was searching for values eigen-.
But the boundary conditions
surpassed all his wishin's
and wouldn't let bygones be bygone.

Oh, trig functions bellow and bark
and Bessel's go bump in the dark.
but Sturm-Liouville creatures
have all sorts of features
and fill up the whole Noah's ark.

ABSTRACT ALGEBRA

We're a bunch of nincompoops.
We can't get a grip on our groups.
We ask, in a frenzy
"Is it Z_n or nZ?"
Guess wrong, so grin and go "Ooops".

This quarter we're all pretty group-y.
It can get pretty meaty and soup-y.
Z_3 order 3
Z_p order p
and D-sub-p order 2p.

(You know how that religious mathematician Kronecker said "God created the integers; the rest is the work of man" -- Well . . .)
"Integers are neat," said God
"both the even and the odd.
"So I'll give you some, Kronecker
"for Christmas or Chanukkah.
"Do you mind if I package them mod?"

Said a group theory pro named McClellan
"We've too many isom and elem
"and homom and autom
"but already bought 'em.
"Perhaps we could try to re-sell 'em."

(Automorphisms)
Some're outer and some're inner.
All're autom's (although it's winter).
Some turn out to be
the mere identity.
But that's good when you're a beginner.

Rub-a-dub, rub-a-dub, bub-bub.
What, pray tell, what, is this hub-bub?
It's subgroups so cool.
They obey the strict rule:
Order-wise, they divide what they're sub ub.

In the kitchen you'll hear lots of slubbering.
In the bathroom you might see a tubring.
But the classroom, I claim
is quite tidy and tame.
All it's got is a subgroup or subring.

(More dialog between God and Kronecker)
”Intergers are great,” said God
“and it’s fun doing sum and prod.
“But I’ve now got some notions
“to tackle the quotients”
and all except K oo’d and ah’d.

“It was nice dividing by bd.
“But now I’m getting more greedy.”
Thus spake our Lord
standing straight at the board
writing Q-bracket-x and Q.E.D.

“One was good for a laugh.
“And two was a treat for my staff.
“But I’ve now got a hankerin’
“to do some tankerin’.
“I’ll start with one-and-a-half.”

“For seven long days labored I
“with integers low, then high.
“But now ‘tis day eight.
“’Tis time to create
“quotients and roots and pi.”

Deep in waters hot, not tepid
feeling rueful but intrepid
our two creators
and integer-traitors
shrugged “Sorry, we just can’t he’p it.”

“Please forgive me,” said Kronecker L.
“I hope you won’t send me to hell.
“But x caught my attention
“and x led to extension”
and the rest we know only too well.

Yes, pity the genius and hero
who fiddled, but not like Nero.
He started with F.
Now he’s got nothing left
for he sold his soul for a zero.

(Primitive Element Theorem)
A fair maid from Alabam-y
was given a double whammy.
But she knew how to mingle
so’s to make it a single
and ace the final exam-y.

(When is F(a) isomorphic to F(x) ?)
If about a we have so spec's
then a might as well be x.
And e and pi
are good a's to try
though it's not quite clear how it checks.

We might grow up to be tax-collectors
or city or country inspectors.
But Chap. 23
ensures that we
will never be angle-trisectors.

(God confesses to Kronecker)
First I couldn't stop at ten.
The I couldn't stop at n.
Then Z and then Q
and the square root of two.
But I've now come to C. Amen.

(Epilogue: by God and Kronecker)
We work together like brothers.
One creates, the other discovers.
And Galleon writes.
And Cohen recites.
And we welcome any others.

PROBABILITY

(Set theory)
Hippity hippity hoppity.
A set can be built through a property
with nouns and verbs
and blubs and blurbs
or any ol' thigamabob-ity.

A set can also be seen
as an element-making machine.
That set will consist
of all s in some list
with commas in between.

A fair maid from North Beelzabub said
"I don't want to get us all upset
"but if s in B
"implies s in C
"then B is of C a subset."

(The empty set)
Here comes phi, consisting of nada
and neither your mada or fada
can fill-er it up
for lunch or for sup
not even with air or with wada.

(Union and Intersection)
The union of 9th and Race
is something we'd care not pace.
But the intersection
leads to detection
of that fabulous Thai lunchplace.

(Universal Set)
Here is one more thigamajig
that we'll need for this whole shindig.
Yup, time for big-U
too big to be true
but not too true to be big.

(Complement)
Whatever set A hath not
is what A-prime hath got.
And their intersection
defies detection
but their union is a lot.

(Cartesian Product)
Let A be of women a set
and B be of men; then we get
from A-cross-B troubles
all possible couples
providing we know they're all het.

($n(A \cup B) = n(A) + n(B)$)
Here's one more essential point:
If A is from B disjoint
then to give you, I'm glad
permission to add
at the risk of seeming flamboyant.

($n(A \cup B) = n(A) + n(B) - n(A \cap B)$)
And now here's another cool fact
on which you might soon need to act:
You still, my comrade
have permission to add
and orders to also subtract.

(to calculate $P_{n,r}$)
Write down n, and then when you're done
write parentheses n - 1.
The next thing to do:
Write n - 2.
Keep going, but not down to none.

(to calculate $C_{n,r}$)
Here's something that's redder than henna:
our blood when computing $C_{n,r}$
and the sweat and the tears
as we and our peers
do cancelling more than we wanna.

A fair maid named Mary Maltese
got her P's all mixed up with her C's.
She said "I do figure
"the P's must be bigger
"but hey! there are no guarantees."

Said her kissin' cousin Muriel
"I'll give you a little tutorial.
"In case you've forgotten
"the C's, on the bottom,
"sport an extra r-factorial."

(Mississippi & Co.)
And now here's a new little game:
permuting when some are the same.
Permuting these some
will feel sort of numb
although there might still be some pain.

(Reality Check)
As these poems you so dutifully edit
keep one thing in mind, to my credit:
I realize that pain
does not rhyme with same.
But it will if you're less anal-headed.

(Binomial Formula)
I'm sure you've a loyal FOIL-er
and I won't be a FOIL-er spoiler.
But when n, for its britch
gets too big, you should switch
and be to this formula loyaller.

(Outcomes)
When you do an experiment-y
the possible outcomes are plenty.
And it's kind of fun
to find out which one.
(It's a good one, Deo volente.)

(Sample Space)
And now let us go with the flow.
Take the set of all outcomes, just so.
Yep, time for big-S.
Yes! yes! yes!
Don't you dare say no!, no!, no!

(Events)
Here's something we'll use infinitum:
Events (We'll both say 'em and write 'em.)
And to poet-ize:
Size-wize, they lies
between the two previous items.

(Another Reality Check)
As these poems you all superintend
allow me to make some amends.
I know that "they lies"
is bad grammar, you-guys.
But what's one s among friends?

(Probability at last)
We can now introduce big-P.
It means probability.
P of S is one.
P of phi is none.
In between lies P of E.

(Probability of Disjoint Unions)
Said a dude with a shrewd attitude
“If events doth each other preclude
“then it’s easy to tune in
“the prob of their union
“providing we’re in the right mood.”

($P(E \cup F) = P(E) + P(F) - P(E \cap F)$)
The above should inspire deja vu.
Take the rule from 6.2
change the n to P.
It seems clear to me.
I hope it seems clear to you.

($P(E') = 1 - P(E)$)
For the prob of E’s compliment
take the prob of that E-event
subtract it from one
and you’ll be all done.
(You can give it up for Lent.)

If the prob of each outcome you do know
then by Jupiter!, Jove! and by Juno!
the sum of them all
will happen to fall
extremely close to uno.

($P(E) = n(E) / n(S)$)
And now here’s our big opportune
to do what we’ve always been doin’ --
permute and combine
but twice this time
on this beautiful late-afternoon.

(One More Reality Check)
As these poems you so dutif’ly edit
keep one thing in mind, to my credit:
I know that time
and combine don’t rhyme
but I’m not overly poetic.

(Expected Value)
First, the possible values collect.
And then, to their prob’s connect.
The grand finale:
Expected val.
What else could we hope to expect?

(Variance and Standard Deviation)
Here’s a question a trible mischevious:
Are the deviations devious?
The answer, I’m ‘fraid
can’t be bought at Rite Aid.
Perhaps we can get it at CVS.

(Binomial Distribution)
n trials, each a yes or a no.
We want x yes's, just so.
The prob of that mix
involves n-choose-x
and the p's and the q's also.

(The Last Reality Check, promise...)
As these verses you classify
as to whether the rhymes apply
you'll notice that mix
does not rhyme with x.
The same would be true of y.

(Some farewell wisdom)
So now that we've had quite our fill
here's a life-lesson from my quill:
The prob's, woebetide
won't be on our side
but the possibilities will.

STAT

(Intros)
I promise, the mean isn't mean.
Nor the deviation obscene.
Nor is any statistic
overtly sadistic.
It hurts, I'll give you morphine.

"The trouble with a kitten is that
"eventually is becomes a cat."
And the trouble with Probability is that
eventually it becomes Stat.

(Summary)
Too big is a populace
to get all the info in place.
So we gather a sample
(one that is ample)
and infer 'til we're blue in the face.

(Data-Values)
You write them all down on a sheet.
(You try hard to make them look neat.)
Some will be qual-
and some will be quant-
and ne'er the twain shall meet.

(Reality Check)
As that last you so dutif'ly edit
keep one thing in mind to my credit:
I know qual- and quant-
(although I might want)
don't rhyme -- still, I'm glad I said it.

Continuous, maybe disrete
in inches, in yards, or in feet
they're numbers all
some big, some small
and some of them dare to repeat.

(for a Population)
The size is denoted big-N.
The mean is small-mu, and so then
for standard dev, sig-
ma (little, not big)
or so we would recommend.

(for a Sample)
Small-n, for the much smaller size
(How could we proceed otherwise?).
Then take the wild guess
of X-bar and small-s
or so I would strongly advise.

(Histograms)
The small x's, if you please
are the values that big-X doth seize
and the various y
so high and so spry
are the relative frequencies.

(A non-limerick)
The less-then probs are truly a breeze
for you and your TI83's.
And the more-than probs, I'm happy to say
are a mere subtract-from-one step away.
And last but not least are the probs between.
TWO steps away and pretty routine.
So get it straight forevermore
and you'll do just fine in Chapter 4.

Three cheers for the theorem of Bayes!
We'll cheer for the rest of our days.
We'll get P-D-T
and P-T-D
as we shout out our hip hp hoorays.

The theorem of Bayes is fun.
And it's certain to get the job done
with sensitivity
and specificity
and some help from P-D and one.

(Binomial distributions)
There's one for each n and each p
in this functional big family.
n, for our files
is the number of trials
and p is the rel- frequency.

(to find P(X(n,p) = x) -- a non-limerick)
Second VARS will get us flowing.
0 gets us where we're going.
n, a comma, p, another
x -- and then it's almost over.
Just make sure that you don't miss
hitting end-parenthesis.
And if you want to see this through
ENTER is the thing to do.

(to find P(X(n,p) ≤ x))
This is something you should love
since it's exactly as above.
Only once diverge the path:
Instead of 0, ALPHA MATH.

(Normal random variables)
The standard normal's a wonder.
At the ends it's tossed asunder.
Its mu is none.
Its sigma is one
which is also the area under.

The curve is an exponential
but knowing that isn't essential.
The 83+
is enough for us
to get a good job at Prudential.

(Non-standard normals)
These curves are all wondrous, too.
But they've got different sigma and mu.
Mu is the head
and sigma's the spread
but the area's one, never two.

(Continuous random variables)
It isn't the curves so grandee.
It's the areas under, you see
that do the fine job
of giving the prob
that X lies between a and b.

(Fuzzy Central Limit Theorem)
Honest, this isn't a scam
that the top of the histogram
(and I do not err)
for "most" X, yes sir
is a normal curve, yes ma'm.

(TI83+ Keystrokes for find Cumulative Prob's) (Warning: Non-limerick)
If this, instead of Earth, were Mars
we might hit VIRTH instead of VARS.
If Earth were square instead of round
we'd arrow up instead of down.
And if we meant "Return to Sender"
what we hit would not be ENTER.
If our goal were finding z
the next four strokes just wouldn't be.
And if we didn't know which z
we wouldn't hit it, natur'ly.
And if we want to poof this venture
we'll forget to punch in ENTER.
But since none of these is true

we'll do just fine with what we do.

(TI83+ Keystrokes for finding z when we know the cumulative prob up to z)
This might be something we all love
since most of it is like above.
Just, if we think it's much too nice
we'll arrow once instead of twice.
And if we're masochists, indeed
we'll do those four strokes we don't need.
And if we didn't know which P
we wouldn't hit it, natur'ly.
And if we were intent-preventers
we'd foget to punch in ENTER.
All in all, it works out great
with cause, effect, and help from fate.

(finding cumulative prob's of non-standard normals)
(If you're wize / you'll standardize.)
Step One. Just take x minus mu
divide by the sigma. Step two:
Dig into your pocket
(or where'er you stock it)
and with your TI rendevous.

(Central Limit Theorem)
If, ladies and fine gentlemen
we take all the samples, size n
then the set of thier means
fulfills all our dreams
for n at least three times ten.

(Confidence Intervals for the Population Mean)
We can never exactly find mu.
That goes for approximate, too.
But the more the dissent
the more confident
we can be; that's the best we can do.

(Calculating Confidence Intervals with the TI83+)
We start with STAT arrow-right twice
and then chose the Z int device
next, put in our data
some this-a, some that-a
and CALCULATE makes it precise.

(Hypothesis Testing -- H-sub-A / leads the way.)
For this very significant lesson
H-A is the thing to obsess on.
H-nought is there
more for the flare.
It makes things a little more pleasant.

(Various “tails”)
“A one-l lama is a priest.
“A two-l llama is a beast.
“And I would bet a silk pajama
“there isn’t any three-l lllama.”

A one-tailed test is a bombshell.
A two-tailed test is something else.
And I would bet a treasure chest
there isn’t any three-tail test.

(The Test Statistic)
Though its various symbols encumber
it really is only a number.
On top goes the gap
on bottom more crap
making us dumber and dumber.

(Using the TI83+ for Hypothesis Testing)
From our trusty TI83
we squeeze out the value of P.
If alpha is steeper
H-A is a keeper
unless further tests disagree.

Yes, trust our trusty one.
It says do, consider it done.
And if P is foremost
H-A is toast.
(We can eat it with cinnamon.)

We start with STAT arrow-right twice
and then choose the proper device
next, put in our data
some this-a, some that-
and CALCULATE gets it precise.

Said a Prob/ Stat whiz named Ms. Bertie
“The CLT is less sturdy
“so instead of the z
“we use little t
“whenever our n’s less than thirty.”

Guiness had said that he cudent
so Gossett decided he wudent
and so little t
is more famous than he
and bears the proud name of Student.

“Difference between” implies two
and that will affect what we do.
There should be two x
(on the screen and the text)
I think that’s a pretty good clue.

Proportions are good to make friends with.
No sigma or s to contend with.
Just n and p-hat.
It makes for good Stat.
So that's what I've chosen to end with.

(Farewell)
If the pain persists on morphine
try an anti-histamine
or caffeine, or saline
or dentine, or codeine
or an anti-statistics vaccine.

TOPOLOGY

We all know the concept of set
but not of topology yet.
The latter's a class
of the first, but alas
there's more to the story than thet.

E.g., it is quite necessary
that they all be in X, solitary
and that intersects, fin-nite
also be in it
and union, arbitrary.

(Accumulation points)
These are points which a set seems to touch
but not, we observe, very much.
They can get just as close
as your eye or your nose
but never as close as your guts.

(Definition of continuity)
No epsilon-delta in this
since numbers don't even exist.
No fractions, decimals,
infinitessimals.
Instead, we must find a new twist.

Points jump from the old to the new
as the sets they comprise also do.
And so, in this scrimmage
the f-inverse image
of each V must be some U.

Tau from tau.
That's the law.

(Base for a topology)
Hee-haw! Hee-haw! Hee-haw!
A base is a sub-class of tau
whose various u-
nions hit through and through
though it itself doesn't at all.

(Sub-base)
Rub-a-dub, rub-a-dub, dub.
A sub-base is not always sub.
But it is a base
in certain ways
so we welcome it into the club.

If X is discrete (tau not slight)
we claim continuity bright
for any old f
from X (on the left)
no matter what Y's on the right.

And here's something equally deft:
If Y is discrete (not bereft)
then open, we cite,
 is Y on the right
no matter what X on the left.

(Homeomorphisms)
Oh, bring out the violins
for here's where the drama begins:
Whither goeth the first
(for best or for worst)
doth goeth the second -- they're twins.

(Metric spaces)
These spaces we truly adore
'cause they provide numbers galore
for us to grab onto
whenever we want to
(just like we used to before).

(Hilbert space)
Square-summable sequences come
with a metric involving square-sum.
And it's isomet-
to a proper subset
which might seem surprising to some.

(first countable)
This means something's countable, yes.
But what? Well, each point must possess
a base local
that is countable
(perhaps more than one, but not less).

(second countable)
Again something's countable, true.
But this time just ONE thing will do
for this kind of space:
one countable base.
I promise, we will not need two.

(separable spaces)
They're countable, in some weird sense
'cause some countable subset is dense
thus reigning them in
agin and agin
making things rather intense.

A fair maid named Mary Michelle
was smitten with Heine-Borel
and her cousin Dinah
liked Borel-Heine.
They got on exceedingly well.

A net is a sort of jet-set.
There's no fish that it cannot get.
With an epsilon-pole
it reaches its goal
and manages not to get wet.

(totally bounded)
Hippety, hippety, hoppety!
A set which possesses this property
is finite, of sorts
(by all sound reports)
though by thinking a trifle sloppity.

(The Separation Properties --- Seeya later, separator...)

Heigh-ho! Heigh-ho! Heigh-ho!
Three cheers for our trusty T-oh.
It separates points
all over the joint --
really just one of them, though.

T-one is a nifty go-getter.
It's a tasty trusty trend-setter.
Yep, it's very hep
the way it doth sep-
and it separates them better.

Yahoo! Yahoo! Yahoo!
It's time for our trusty T-two.
It deftly gets
TWO open sets.
Yes, that's what we trust it to do.

(regular spaces)
Here comes a whole different name
playing a whole different game.
It dares to make bets
on points and closed sets
putting the others to shame.

(normal)
This guy does a little duet.
It can take on any two sets
that are diagnosed
as being closed.
It's the best separator yet.

SEPARATION STEW
We learned T-oh
to and fro.
And then T-one
was lots of fun.
We did T-two
fro and to.
Went through T-three
with glitz and glee.
And T-three-and-a-half
gave us a laugh
until T-four
walked in the door.

I'm sure T-five
would really jive
and with T-six
we'd get our kicks.
Lucky T-seven
would be pure heaven.
Likewise T-eight
would be just great.

T-alept-null
would be wonderful.

But we stop at four.
There is no more.

(In any normal space, any two closed sets can be "separated by a continuous function".)
How very superb and eclectic
to deal with this layout so hectic.
With mere open sets
cool Urysohn gets
all these numbers with nary a metric.

FERMAT'S LAST THEOREM PROVEN

Fermat said the proof was too large
to fit in the right or left marg-.
True, back of the paper
or proof made to taper
might help, but he said, "I'm in charge".

Now, Wiles didn't mind paper waste.
In fact, it was true to his taste
to use up whole reams
to realize his dreams
and he crossed out instead of erased.

Fermat was all snickers and smiles
as he smugly stayed clear of the aisles
and he thought "they'll be glum
"but that proof will succumb
"though it's going to take quite a-Wiles".

A COMPUTER NON-LIMERICK

A one-r erol is a strain.
A two-r errol is a pain.
But we would be in greater peril
if there were a three-r errrol.